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MODELING TAMPING RECOVERY OF TRACK GEOMETRY USING THE COPULA-BASED APPROACH

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ABSTRACT

Assessing and maintaining track geometry within acceptable limits are key components of railroad infrastructure maintenance operations. Track geometry conditions have a significant influence on rider comfort and safety. To maintain the ride quality and safety of the track, maintenance activities pertaining to track geometry, such as tamping, are performed. Tamping enhances the track geometry quality but fails to return the track geometry to an as-good-as-new condition. Majority of studies have evaluated tamping recovery using deterministic techniques, which assume that tamping recovery is dependent on the track geometry quality prior to tamping. However, they fail to capture the uncertainty of the recovery values. Probabilistic approaches are increasingly being used to account for the uncertainty but fail to model the underlying dependence between the variables, which may exhibit nonlinear dependences such as tail or asymmetric dependence. To accurately model the tamping recovery phenomenon, this research employs the copula models in combining arbitrary marginal distributions to form a joint multivariate distribution with the underlying dependence. Copula models are used to estimate the tamping recovery of track geometry parameters such as surface (longitudinal level), alignment, cross level, gage, and warp.

Keywords: Copulas, track geometry, tamping, railroad maintenance, correlation analysis, concordance measures

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INTRODUCTION

Railroad tracks deteriorate with age and usage (tonnage) with decreasing performance over time, which may eventually lead to failure. Railroad infra- structure components often have a service life of more than 30 years justifying the need for an optimal long-term maintenance strategy. Due to budget restrictions and high logistical cost constraints, rail-roads plan most track geometry maintenance activities up to a year in advance (Quiroga et al. 2012, Soleimanmeigouni et al. 2016, and Caetano and Teixeira 2016).

Track geometry is a key feature of railroad construction (Esveld 2001, Khouy 2013). The condition of track geometry is important for various reasons. Riding comfort and safety (risk of derailment) are dependent on the condition of track geometry (Quiroga et al. 2012). A well-maintained track geometry not only guarantees ride comfort and safety but also increase the life of the track and track availability for train operation. Thus, maintenance of track geometry is imperative in relation to cost reduction and availability of tracks (Famurewa et al. 2016). Furthermore, deterioration of many other track components is closely linked to the condition of track geometry (Khouy 2013 Jovanovic 2004).

Maintenance activities of track geometry are regularly conducted in order to maintain the track geometry condition to achieve good riding quality and safety (Miwa 2002). These activities such as tamping, stone- blowing, and ballast undercutting are conducted to control track deterioration and recover damaged track sections to operable conditions. They enhance the quality of track geometry but fail to return the track geometry to a good-as-new condition (Soleimanmeigouni et al. 2016). If prognostic (predictive) tamping strategies are to be employed, there is the need to know beforehand the effectiveness of tamping, which can be evaluated by the amount of improvement or recovery in the condition of track geometry (Famurewa et al. 2013).

Majority of studies have evaluated tamping recovery using deterministic techniques such as linear regression models and have assumed that tamping effectiveness is mainly dependent on the quality of track geometry prior to tamping. However, in most i cases there exists a high degree of uncertainty due to high variation in the restoration values after tamping even for similar track geometry condition. This variation is even higher at the end of the life-cycle than at the beginning. For this reason, probabilistic or stochastic techniques have been employed to cater for this variation by assuming the recovery value after tamping is a random variable with a given probability distribution (Soleimanmeigouni et al. 2016).

Furthermore, most tamping recovery models do not take into account the underlying dependence between the tamping recovery values and the influencing factors such as track geometry condition before tamping. In this paper, a copula-based approach is employed, which takes into consideration the various forms of dependences by allowing for the separate modeling of the arbitrary marginal distributions and the dependence structure that are subsequently combined to form a joint distribution with the underlying dependence.

TRACK GEOMETRY MAINTENANCE AND RECOVERY

Track geometry

Track geometry may be defined as the three-dimensional geometry of track layouts and related measurements used in design, construction, and maintenance of railroad tracks. To identify defects prior to their development beyond acceptable standards, track geometry condition is regularly evaluated during track inspection (Caetano and Teixeira 2016, Caetano and Teixeira 2015). Track geometry is influenced by climatic conditions, traffic conditions such as loads and speed, construction materials and techniques, as well as maintenance history (Audley and Andrews 2013).

Quality of track geometry

The quality of track geometry can be defined as the "assessment of deviations (excursions) from the mean or designed geometrical characteristics of specified parameters in the vertical and lateral planes which give rise to safety concerns or have a correlation with ride quality" (Khouy 2013). The track geometry condition can be assessed by the standard deviation (SD) over a specified length, mean value or extreme (peak) values of isolated defects of the track geometric par- ammeters (Khouy 2013, Vale et al. 2012). The main geometric parameters used to evaluate the quality and irregularity of track geometry include surface (longitudinal level or vertical alignment), alignment (horizontal alignment), gage (gauge), cross level (cant), and warp (twist). Surface, cross level, and warp are vertical geometric parameters, whereas alignment and gage are horizontal geo- metric parameters.

The deterioration of track geometry is often evaluated by the irregularities or defects of these parameters: surface defects, horizontal alignment defects, cross-level defects, gage deviations, and warp (track twist) deviations. Infrastructure managers often com- bine these defects into a track quality index (TQI) as a representative measure of the different track geometric parameters, and the index is quantified as a function of the SDs of each irregularity and allowable train speed. However, the SD of short wavelength of the surface defect is still regarded as the decisive criterion for maintenance decisions Caetano and Teixeira 2013, Andrade and Teixeira 2012).

Surface and alignment can be defined as the track geometry of railroad track centerline projected onto longitudinal vertical and horizontal planes, respectively. The surface parameter is considered to be the most representative of the track quality (Audley and Andrews 2013). It is the main factor for determining the expenses of track maintenance and often triggers the need for maintenance intervention (Khouy 2013). It is the geometric parameter, which significantly affects rolling stock and the track dynamics in the vertical direction (Vale et al. 2012). Surface irregularities can be defined as the vertical geometric deviation measured in inches from the rail top on the running surface to the ideal mean line of the longitudinal profile. Shortwave surface defects have been found to recover very well during tamping. Experimental studies have verified a linear dependence between SD of surface irregularities and accumulated tonnage. Despite surface being the most prominent parameter, disregarding the other parameters during the evaluation of track geometry condition may result in erroneous assessment leading to ineffective maintenance planning (Caetano and Teixeira 2016, Soleimanmeigouni et al. 2016, Andrade and Teixeira 2012).

Gage is the distance between two rail heads at right angles to the rails in a plane 5/8" below the top of the rail head. This differs in Europe where the gage is measured 14 mm (0.55") below the running surface. Gage variation along with alignment has been found to play important roles in the operational quality of the railroad track substructure. Warp (twist) is a measure of the cross-level variation. Warp can also be defined as the algebraic difference between two cross levels taken at any two points within a specified chord length and is usually expressed as the gradient between the points. Cross level on the other hand is the difference in elevation between the adjacent running rails computed from the angle between the running surface and a horizontal reference plane. Warp is a crucial factor that is considered during derailment risk assessment and thus must not be ignored during the evaluation of track geometry (Soleimanmeigouni et al. 2016).

There is the need for regular inspection or monitoring of the condition or quality of track geometry using track geometry inspection cars. Track geometry inspection cars assess track irregularities using both an inertia measurement system and an optical system. The vertical and lateral deviation of the track is computed for consecutive 1-foot measurements by means of recorded vehicle accelerations measured by an accelerometer. The sampling interval differs in Europe where track parameters are usually measured at 25 cm (0.82 ft) intervals.

Tamping

Tamping is the main maintenance activity employed to restore the track geometry condition and is one of the most essential yet costly track maintenance activities (Caetano and Teixeira 2016, Wen et al. 2016). Tamping rectifies the track geometry deviations such as incorrect surface profile (vertical deviation) and incorrect alignment (lateral deviation) by rearranging and compacting the ballast (Audley and Andrews 2013, Khouy 2016).

Tamping can be executed either mechanically or manually (Audley and Andrews 2013) and involves heavy machinery and substantial labor resources (Caetano and Teixeira 2016). Tamping operations can be performed as either preventive or corrective maintenance (Khouy et al. 2012). Corrective tamping is performed to rectify isolated defects, whereas preventive tamping can be performed at stations, turnouts (switches) and crossings, and open lines. These two kinds of tamping procedures are planned in different ways (Wen et al. 2015). Tamping can also be classified into complete and partial tamping procedures. Complete tamping intervention is executed on the entire length of track section, whereas partial tamping is carried out on a fraction of the segment. Complete tamping and partial tamping have different effects on the track geometry condition. Thus, separate analysis of these kinds of interventions can result in a drastic decrease in the variation of recovery values of track quality after tamping (Soleimanmeigouni et al. 2016a, SoleimanmeigouniI et al. 2016b).

Tamping results in a significant decrease in the track geometry irregularity measurements and alters the track deterioration (Soleimanmeigouni et al. 2016). Tamping also has a significant influence on the effective capacity of a railway network as a result of its distinct needs such as track possession duration, track quality demand, scheduling constraints, and heavy equipment utilization. Thus, it is important to optimize the scheduling of this maintenance task (Famurewa et al. 2013, Gustavsson 2015). However, the execution of tamping more often is not optimally planned. Tamping are at times performed at very low (SD) levels and thus are not influenced by travel comfort (Khouy et al. 2012). Hasty tamping may result in shorter life cycle and track design capacity may not be attained given the ineffective tamping procedures (Famurewa et al. 2013, Quiroga et al. 2012).

Tamping recovery

The recovery in track geometry condition may be dependent on several factors including track quality prior to tamping, frequency of previous tamping operations (maintenance history), subsurface (ballast) conditions, tamping procedure, age of track components, operational speeds, and human factors (Famurewa et al. 2013, Audley and Andrews 2013). The dominant factor that influences tamping efficiency or tamping recovery is the track condition just before tamping. The recovery of the SD of the surface profile depends on the track geometric quality just prior to maintenance according to the Office for Research and Experiments of the UIC. The higher the SD of the surface profile, the higher the variability of the track recovery (Vale et al. 2013).

Tamping recovery is dependent on previous tamping procedures since tamping has a damaging effect on the ballast (the tamping machine arms crush the ballast particles), which is the major factor of track stability. This leads to the resultant quality in the current tamping being lower than the resultant quality of the preceding tamping (Wen et al. 2016). Tamping recovery also reduces with increasing number of accumulated tamping interventions due to the ballast deterioration with traffic loads as well as the ballast damage due to successive tamping procedures (Caetano and Teixeira 2016). Tamping efficiency decreases with increase in ballast service life leading to a reduction in the durability of track quality and increased frequency of tamping to maintain track condition at acceptable standards (Zhao et al. 2006).

There are two main approaches of modeling restoration (or recovery) after tamping, namely deterministic or probabilistic (stochastic) approaches. The choice of methodology to employ should be chosen basedon the degree of uncertainty in the recovery values after tamping. In deterministic techniques, tamping recovery is directly evaluated in relation to the influencing factors such as track quality prior to tamping, the operational speeds, and maintenance history. The model parameters are treated as unknown constants with uncertainty incorporated using confidence intervals. Majority of studies have evaluated tamping recovery using deterministic techniques such as linear regression models and have assumed that tamping effectiveness is mainly dependent on the quality of track geometry prior to tamping. Linear regression models are highly popular due to their simplicity and have been employed in the development of track geometry maintenance models and optimization scheduling models (Soleimanmeigouni et al. 2016a, Soleimanmeigouni 2016b)

Miwa (2002) and Oyama and Miwa (2009) applied linear regression restoration models to predict the maintenance effectiveness of tamping with the amount of recovery dependent on the track condition prior to tamping. Their restoration models were combined with an exponential smoothing degradation model, which were subsequently used in developing an optimizationtrack maintenance scheduling model. Andrade and Teixeira (2012) employed linear tamping restoration models as well as a linear track deterioration model, which were subsequently used in the development of a bi-objective model to optimize planned maintenance and renewal activities related to track geometry. Vale et al. (2012) employed linear tamping restoration model as well as a linear track deterioration model, which were subsequently used in the development of a mathematical maintenance model (formulated as integer (mixed 0-1 linear programming), which optimizes tamping operations in ballasted track as preventive maintenance.

Meier-Hirmer et al. (2009) developed a maintenance strategy model comprising three sub-models namely an intervention efficiency model, a gamma process track deterioration model, and a maintenance cost model. This model was used to establish the long-term costs of various maintenance strategies and optimize these costs based on various parameters such as intervention threshold or inspection interval. The authors observed that the maintenance efficiency or recovery appeared to be normally distributed and employed linear regression to characterize the intervention benefit, which was assumed to be dependent on the deterioration prior to intervention. Famurewa et al. (2013) developed an empirical regression model for recovery after tamping intervention based on previous (longitudinal level) data on examined routes. The empirical recovery model was combined with an exponential track degradation model to optimize the tamping intervention schedule through the minimization of the total intervention cost particularly the track possession cost.

Wen et al.2016) evaluated the tamping recovery based on both the track condition before tamping and the frequency/number of previously performed tamping procedures. This restoration model was subsequently employed in a mixed integer linear programming (MILP) model formulated for the scheduling optimization of preventive condition-based tamping through the minimization of net present costs considering several factors. Caetano and Teixeira[3] evaluated the effect of the age of track sections (segments) operations on tamping recovery by comparing renewed sections (ages of approximately 10 years) and nonrenewed sections (approximately 20 years). Despite the variation in the deterioration rates of track geometry due to loss of tamping effectiveness, the average number of maintenance tamping procedures was found to be greater in older track sections. This is similar to the findings by Audley and Andrews (2013).

Khouy et al. (2012) evaluated the effectiveness of tamping by examining the track condition (longitudinal level) before and after tamping, which was subsequently categorized using a tamping intervention graph into bad, good, or excellent in relation to the level of improvement in track condition after maintenance. A large proportion of the sections were found to be either in the good or bad category. Due to the high variation in the recovery observed, factors such as the effect of ballast age on tamping efficiency were evaluated. However, no clear effect of ballast age was noted contrary to the findings by Caetano and Teixeira (2015) and Audley and Andrews (2013). Soleimanmeigouni et al. (2017) proposed two-level piece- wise linear model to characterize the recovery and deterioration of track geometry with possible spatial dependencies within deterioration parameters captured using autoregressive moving average models. Multivariate linear regression was employed to tie various explanatory variables with response variables such as recovery values and changes in the deterioration rates after tamping. Tamping recovery was dependent on both track condition before tamping and tamping type (partial or complete) with the inter- action effect between the two covariates also considered.

Linear regression models are highly popular due to their simplicity. However, they assume linear dependency and assume normality of the random variables and joint distribution. Non-normality transpires in various forms: non-normality of marginal distribution of some variables and in some instances multivariate non-normality of the joint distribution of a group of variables despite normal marginal distributions of all the individual variables (Yan 2006, Attoh-Okine 2013). Furthermore, in most cases there exists a high degree of uncertainty in recovery values even in instances where track quality is identical prior to tamping, which cannot be accounted for using deterministic techniques. For this reason, probabilistic techniques have increasingly been employed to cater for this variation by assuming the recovery after tamping is a random variable with a given probability distribution. A unique distribution for the recovery values after tamping is selected given a group of influencing variables with the parameters (or measures) of the distribution assumed to be a function of the inputs (Soleimanmeigouni et al. 2016b).

Quiroga and Schnieder (2012) developed a simulation approach for modeling the recovery and degradation of track geometry. The stochastic model statistically characterizes the phenomena given the historical data and employs Monte Carlo method to attain simulated process realizations. The tamping recovery was assumed to be dependent on the number of accumulated tamping interventions. The track quality (longitudinal mean deviation) after tamping was assumed to be log-normally distributed stochastic variable dependent on the number of accumulated tamping interventions. It was observed that the variance of the track quality (longitudinal mean deviation) after tamping increased with greater number of accumulated tamping interventions. It was also observed that the deterioration rate (quality loss rate) increased considerably after each tamping intervention. Quiroga et al. (2012) combined the Monte Carlo simulation approach developed by Quiroga and Schnieder (2012) and a heuristic algorithm for maintenance intervention planning to evaluate the optimization of two maintenance strategies namely adaptive (dynamic) and constant intervention thresholds.

Audley and Andrews (2013) evaluated the effect of tamping on the degradation of track geometry condition taking into consideration two probability distributions, which characterize the track quality for periods between tamping. Firstly, the authors analyzed the distributions of times for the track geometry to degrade to specified states or levels of performance following tamping given the line speed and the maintenance history. The two-parameter Weibull distribution was found to best model the times to degradation despite the better fit of its three-parameter counterpart since the extra parameter (location parameter or failure-free parameter) pro- vided a better fit but no physical reason to justify a nonzero location parameter. Results of the analysis corroborated the theory that tamping damages the ballast and results in faster deterioration of the track geometry, which was evident by the reduction of the characteristic life parameter with the frequency of tamping interventions. Additionally, it was observed that the more the track geometry degrades, the greater the rate of degradation, which was evident by the increase in the shape parameter with the measurement of track quality (SD of the vertical alignment). Secondly, the authors analyzed the quality of track geometry after intervention. Despite the three-parameter log-normal distribution having the best-fit, two-parameter log-normal distribution with a slightly lower fit was selected due to its ease of use to model the recovery values after tamping (probability of achieving the track quality condition after tamping) given the operational speeds and maintenance history. Tamping efficiency was found to decrease with increasing number of accumulated tamping interventions, which provides further proof that tamping damages ballast. Tamping efficiency was also found to reduce with increase in the operational speed.

Soleimanmeigouni et al. (2016) evaluated the effect of tamping on several (different) track geometric parameters such as surface (longitudinal) level, alignment, and cross level (cant) analyzing both the tamping recovery as well as the change in the deterioration rate after tamping (due to tamping). A probabilistic model was used to model tamping recovery of the geometric parameters, which was assumed to be dependent on the track geometry condition prior to tamping. The deterioration of track geometry was modeled using linear regression and Wiener process. The recovery values of the cross level (cant) and alignment were assumed to follow a three-parameter log- normal distribution while the recovery values of the surface profile were assumed to follow a three- parameter Weibull distribution. Tamping was found to have a negative effect (impact) on the deterioration rate with the increase in the degradation rate evident by the observed increase in the regression slope and drift coefficient of the Wiener process. Complete and partial tamping interventions were also clustered and examined separately since they have different effects on the track geometry condition. Complete tamping interventions were found to have a considerably greater effect on the track geometry condition compared to partial tamping. Additionally, a linear correlation analysis conducted showed a moderate dependence (correlation) between the recovery of sur- face (longitudinal) level and that of the cross level (cant) and a weak dependence (correlation) between the surface (longitudinal) level and that of the alignment. However, Pearson's correlation coefficient assumes linear dependence between the random variables and assumes normality of these random variables and their joint distribution. Thus, it will be more appropriate to employ concordance measures which are suitable for measuring both linear and non- linear dependence. These measures are scale-invariant and measure dependence irrespective of the assumed distributions.

In summary, the vast majority of tamping recovery models do not take into consideration the underlying dependence between the variables of interest, which may exhibit tail dependency, asymmetric dependence, and other non-linear dependencies. However, copula-based approaches take into account these nonlinearities by allowing for the separate modeling of the arbitrary univariate marginal distributions and the dependence structure, which are subsequently combined to form a joint distribution with the underlying dependence.

COPULAS

General

Copulas are functions that combine or link multivariate distribution functions to their univariate marginal distribution functions and are thus more flexible than standard elliptical distributions. An *n*-dimensional copula is a multivariate distribution function $C(u_1, \dots, u_n)$ defined on the unit hypercube $[0, 1]^n$, with *n*-random variables as uniformly distributed marginal (Nelsen 2006, Czado et al. 2012, and Zilko et al. 2016). The copula function assigns a non- negative number to each hyper-rectangle in the unit hypercube. C is a bivariate copula if C: $[0, 1]^2 \to [0, 1]$ and meets the following conditions:

1. $C(u,0) = C(0,v) = 0$ for any $u, v \in [0,1]$
2. $C(u,1) = u$ and $C(1,v) = v$ for any $u, v \in [0,1]$
3. $C(u_2,v_2) - C(u_1,v_2) - C(u_2,v_1) + C(u_1,v_1) \geqslant 0$
   $\forall \quad 0 \leqslant u_1 \leqslant u_2 \leqslant 1$ and $0 \leqslant v_1 \leqslant v_2 \leqslant 1$

The copula approach via Sklar's theorem (Sklar 1959) permits the separation of the multivariate distribution into univariate margins, and the dependence structure, which is modeled via the copula function without loss of information (Dalla et al. 2016). Sklar's theorem offers the link between univariate marginals and copula to the multi- variate joint distribution. Sklar's theorem states that for any *n*-dimensional distribution function with given marginals $F_1, \dots, F_n$, there exists an *n*-dimensional copula $C : [0, 1]^n \to [0, 1]$ such that for all $(x_1, \dots, x_n) \in R^n$

$$F(x_1, \ldots, x_n) = C\{F_1(x_1), \ldots, F_n(x_n)\} \quad (1)$$

holds. C is unique if each $F_i\ (x)$ is continuous; otherwise it is uniquely determined by the product of their ranges (Range of $F_1 \times \ldots \times$ Range of F_n)

Sklar's theorem offers a useful means of constructing copulas given the marginals F_1, ..., F_n such that

$$C(x_1, \ldots, x_n) = F(F^{-1}(x_1), \ldots, F^{-1}(x_n)) \quad (2)$$

If F is absolute continuous, the copula density c is well defined and can be written as

$$c(u_1, \ldots u_n) = \frac{\partial^n C(u_1, \ldots u_n)}{\partial u_1, \ldots \partial u_n} \quad (3)$$

The density f of the multivariate distribution F given the copula density c can be expressed as

$$f(x_1, \ldots x_n) = c\{F_1(x_1), \ldots, F_n(x_n)\} \prod_{i=1}^{n} f_i\ (x_i) \quad (4)$$

There are two popular classes of copulas namely elliptical copulas and Archimedean copulas with a third less common class called extreme-value copulas (Yan 2006).

Elliptical copulas

Elliptical copulas are copulas of elliptical distributions. The two most common elliptical copulas are the normal or Gaussian copula and the Student's t copula, which are related to the multivariate normal and multivariate Student's t distributions, respectively. Both of these copulas are tail-symmetric; however, Student's t copula has tail dependence, whereas Gaussian copula has no tail dependence. Properties of bivariate elliptical copula families including parameter range, Kendall's tau, and tail dependence is given in *Table 1*. Elliptical copulas are directly obtained by the inversion of Sklar's theorem and thus can be expressed in the form

$$C(u_1, \ldots, u_n) = F(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n)) \quad (5)$$

Archimedean copulas

Archimedean copulas are constructed by means of a complete monotonic function without the need for distribution functions or random variables (Yan 2006). Common one-parameter Archimedean copulas include Clayton, Gumbel, Frank, and Joe copulas. Common two-para- meter Archimedean copula families include Clayton– Gumbel (BB1), Joe–Gumbel (BB6), Joe–Clayton (BB7), and Joe–Frank (BB8), which are more flexible. Archimedean copulas can be expressed as

$$C(u_1, \ldots, u_n) = \varphi^{-1}(\varphi(u_1), \ldots, \varphi(u_n)) \quad (6)$$

where the generator of the copula, $\varphi:[0,1]\rightarrow[0,\infty]$ is a continuous strictly decreasing convex function such that $\varphi(0)=\infty$ and $\varphi(1)=0$ and φ^{-1} is its pseudo-inverse which is given as

$$\varphi^{-1}(\nu) = \begin{cases} \varphi^{-1}(\nu) & 0 \leqslant \nu \leqslant \varphi(0) \\ 0 & \varphi(0) \leqslant \nu \leqslant \infty \end{cases} \tag{7}$$

The properties of various one-parametric and two- parametric bivariate Archimedean copulas are given in *Table 2*.

Since copulas are invariant under monotone trans- formations, scale-invariant measures of dependence such as Kendall's tau and Spearman's rho are more suitable for evaluating the degree of dependence. They are both rank correlations and remain unaltered under strictly increasing transformations and evaluate a form of dependence known as concordance, which is the agreement or consistency of two or more sets of rankings (Yan 2006, Nelson 2006).

Nonlinear dependence is usually evaluated using Kendall's tau (Czado et al. 2012). Kendall's tau measures dependence independent of the assumed distribution and thus is suitable when linking various (non-Gaussian) copula families (Dissmann et al. 2013). For Archimedean copulas, the closed-form expression of Kendall's tau is based on the copula-specific generator function, whereas their computation for elliptical copulas are more complicated (Schepsmeier and Czado 2016). The Kendall's tau for various bivariate elliptical copulas and bivariate Archimedean copulas are shown in Tables 1 and 2, respectively.

Other concordance measures include Gini's measure of association, Blomqvist's measure of association (or medial correlation coefficient), and Moran's coefficient (Nelsen 2006 and Dorey and Joubert 2005). There are several advantages of using rank correlations over ordinary product moment correlations such as Pearson's correlation coefficient, which assumes linear dependency and normality of the random variables. These advantages include: they always exist, they are independent of marginal distributions meaning they can take any value in the [1, -1] interval, and they are invariant under mono- tonic increasing transformations of the marginal (Bedford T and Cooke 2001).

As a result of the aforementioned limitations of Pearson's correlation coefficient, concordance dependence measures such as Kendall's tau and Spearman's rho were employed as the measures of dependency in this case study. Kendall's tau was used in evaluating the dependency between the track condition before tamping and tamping recovery values. Furthermore, Kendall's tau and Spearman's rho were used in measuring the dependence between the recovery values of the various geometric parameters. These scale-invariant measures were subsequently compared with the Pearson's correlation coefficient.

Table 1. Properties of the Bivariate Elliptical Copula Families (Brechmann and Schepsmeier 2013).

Elliptical copula	Parameter range	Kendall's tau	Tail dependence
Gaussian/ Normal	$\rho \in (-1, 1)$	$\frac{2}{\pi}\arcsin(\rho)$	0
Student's t	$\rho \in (-1, 1), \nu > 2$	$\frac{2}{\pi}\arcsin(\rho)$	$2t_{\nu+1}\left(-\sqrt{\nu+1}\sqrt{\frac{1-\rho}{1+\rho}}\right)$

Table 2. Properties of the Archimedean Bivariate Copula Families (Brechmann and Schepsmeier 2013).

Name	Generator function	Parameter range	Kendall's tau	Tail dependence (lower, upper)
Clayton	$\frac{1}{\theta}(t^{-\theta}-1)$	$\theta>0$	$\frac{\theta}{\theta+2}$	$(2^{-\frac{1}{\theta}},0)$
Gumbel	$(-\log t)^{\theta}$	$\theta\geqslant 1$	$1-\frac{1}{\theta}$	$(0,2-2^{\frac{1}{\theta}})$
Frank	$-\log\left[\frac{e^{-\theta t}-1}{e^{-\theta}-1}\right]$	$\theta\in\mathbb{R}\setminus\{0\}$	$1-\frac{4}{\theta}+4\frac{D_1(\theta)}{\theta}$	(0, 0)
Joe	$-\log[1-(1-t)^{\theta}]$	$\theta>1$	$1+\frac{4}{\theta^2}\int_0^1 t\log(t)(1-t)^{\frac{2(1-\theta)}{\theta}}dt$	$(0,2-2^{\frac{1}{\theta}})$
Clayton–Gumbel (BB1)	$(t^{-\theta}-1)^{\delta}$	$\theta>0$, $\delta\geqslant 1$	$1-\frac{2}{\delta(\theta+2)}$	$(2^{-\frac{1}{\theta\delta}},2-2^{\frac{1}{\delta}})$
Joe–Gumbel (BB6)	$(-log[1-(1-t)^{\theta})^{-\delta}$	$\theta\geqslant 1$, $\delta\geqslant 1$	$1+\frac{4}{\delta\theta}\int_0^1(-\log(1-(1-t)^{\theta})$ $\times(1-t)(1-(1-t)^{-\theta}))dt$	$(0,2-2^{\frac{1}{\delta\theta}})$
Joe–Clayton (BB7)	$(1-(1-t)^{\theta})^{-\delta}-1$	$\theta\geqslant 1$, $\delta>0$	$1+\frac{4}{\theta\delta}\int_0^1(-(1-(1-t)^{\theta})^{\delta+1}$ $\times\frac{(1-(1-t)^{\theta})^{-\delta})-1}{(1-t)^{\theta-1}})dt$	$(2^{-\frac{1}{\delta}},2-2^{\frac{1}{\theta}})$
Joe–Frank (BB8)	$-log\left[\frac{1-(1-\delta t)^{\theta}}{1-(1-\delta)^{\theta}}\right]$	$\theta\geqslant 1$, $\delta\in(0,1]$	$1+\frac{4}{\theta\delta}\int_0^1\left(-\log(\frac{(1-t\delta)^{\theta}-1}{(1-\delta)^{\theta}-1}\right)$ $\times(1-t\delta)(1-(1-t\delta)^{-\theta}))dt$	(0,0)

All the aforementioned correlation coefficients measure the average dependence over the domain of the variables of interest. Tail dependence is the measure that tries to capture the dependence more locally rather than globally, in the tails (lower and/or upper) of distribution (Lewandowski 2008). It can be defined as the measure of the co-movements in the tails of the distributions of random variables. Let $Y=(Y_1, Y_2)$ be a pair of random variables. The pair is said to be upper tail dependent if

$$\lambda_U = \lim_{\upsilon \to 1} P\{Y_1 > F_1^{-1}(\upsilon) | Y_2 > F_2^{-1}(\upsilon)\} > 0 \tag{8}$$

if the limit λ_U exists. This is the probability that Y_1 reaches extremely large values, given that Y_2 attains extremely large values. Similarly, the pair is said to be lower tail dependent if

$$\lambda_L = \lim_{\upsilon \to 0} P\{Y_1 \leqslant F_1^{-1}(\upsilon) | Y_2 \leqslant F_2^{-1}(\upsilon)\} > 0 \tag{9}$$

if the limit λ_L exists. The lower and upper tail dependence coefficients of various elliptical and Archimedean copula families can be found in Tables 1 and 2, respectively. If the lower and upper tail coefficients differ, the dependence can be said to be asymmetric. Asymmetric dependence is the dependence that is not identical on both sides of a central line or line of symmetry over the domain of the variables of interest. On the other hand, symmetric dependence is the dependence that is identical on both sides of a central line or line of symmetry.

TRACK INFORMATION AND DATA COLLECTION

One mile of track of a Class 1 U.S. railroad was used for the analysis. Inspection data were measured and collected for every 1 foot of track using a track geometry car. The track geometry car records several geo- metric parameters. However, the surface, alignment, cross level, gage, and warp were used for the analysis. The inspection data used in this case study span the years 2014 to 2016 and were generally collected on a monthly basis. Thus, the time period between inspection data before and after tamping interventions was about a month apart.

The inspection data were initially cleaned and pre- processed. Preprocessing of the data included signal realignment and application of a moving average filter to remove random noise or high frequency content in the signals. The SD of each of the track geometric parameters was subsequently computed for track segments with 100 feet of length. The tamping recovery values for each parameter were obtained by computing the difference between the SD of the track geometric parameters before tamping and the corresponding standard deviation after tamping.

ANALYSIS

Marginal fitting

In order to select the best-fit for the marginal distributions for the recovery values after tamping, track quality before tamping and track quality after tamping; the Kolmogorov–Smirnov (KS), Anderson–Darling, and Chi-squared tests were chosen as the goodness-of-fit criteria. These test statistics evaluate how well the data (stochastic variable) follow a specific (an apriori) distribution. The smaller the statistic, the better the distribution fits the given data. In order to select the best distribution, the statistic should be considerably lower than the others, else additional criteria such as probability plots need to be employed. The null hypothesis states that the data followed a specified distribution with the alternative hypothesis stating that the data do not follow a specified distribution. The null hypothesis is rejected if the *p*-value is lower than a significance level of 5%.

The KS test is a nonparametric statistical test of the equality of two probability distributions namely the empirical distribution of the data and a reference probability distribution. The Anderson–Darling test offers more weighting to the tails compared to the KS test.

Copula fitting

The underlying dependence between track quality before tamping and recovery values as well as the dependence of the track quality before tamping and track quality after tamping was characterized using copulas. In order to select the best-fit of bivariate copula that describes the underlying dependence, the Akaike information criterion (AIC) (Akaike 1974) and Bayesian information (BIC) (Schwarz 1978) were used. AIC corrects the log-likelihood of a copula for the number of parameters. AIC is often favored for bivariate copula selection ahead of other alternative criteria such as Vuong (1989) and Clarke (2007) goodness-of-fit tests and BIC. This is as a result of its high performance in simulation analysis and its greater reliability (Dalla et al. 2014, Dissmann et al. 2013).

Prior to selection of the bivariate copula, the Genest and Favre bivariate asymptotic independence test based on Kendall's tau is performed to determine the independence of the pair of variables. The null hypothesis states that the variables are independent and the alternative hypothesis states that the variables are not independent. The independence copula is selected for the pair of variables if the *p*-value of the test is higher than 5% meaning the null hypothesis is accepted.

The pair-copula families considered during the analysis were the *independence copula*, *elliptical bivariate Gaussian (Normal)*, and *Student's t copulas* as well as the single parameter Archimedean copulas such as *bivariate Clayton, Gumbel, Frank, and Joe copulas*. Others include the two-parameter Archimedean copulas such as *Clayton–Gumbel (BB1), Joe–Gumbel (BB6), Joe–Clayton (BB7), and Joe–Frank (BB8) copulas*. The Clayton–Gumbel (BB1) and Joe–Clayton (BB7) permit different nonzero lower and upper tail dependence coefficients.

Rotated versions (90° and 270°) of these Archimedean copulas can be used to fit negative dependences (with the exception of Frank copula that has no rotated version). However, no negative dependences were observed during exploratory analysis so these rotations were not considered during further analysis. This catalog for the implementation of copula family choice addresses a vast range of dependence behavior. Properties of these copulas are found in Table 3.

RESULTS

The marginal and copula fitting results for each track geometry indicator namely surface, alignment, cross level, warp, and gage are reported in this section.

Surface (longitudinal level)

Marginal fitting. The three-parameter log-normal distribution was found to have the best-fit of the recovery values of the SD surface producing the lowest statistic for all three tests namely the Kolomogorov–Smirnov, Anderson–Darling, and Chi-squared tests as shown in Table 4. It also had a *p*-value far greater than 0.05 for KS and Chi-squared tests meaning the null hypothesis that it follows the distribution can be accepted. The closed-form expression for the p-value of the three-parameter log-normal distribution, however, does not exist for the Anderson–Darling test. Audley and Andrews (2013) found the three-parameter log-normal distribution to have the best-fit of recovery values of the SD surface profile but employed its two-parameter counterpart due to its ease of use. The two-parameter log-normal distribution has been used to model the recovery values of the surface profile (longitudinal level) by several researchers (Quiroga and Schnieder 2012, Audley and Andrews 2013, Quiroga et al. 2012). However, in this case study, the three-parameter log-normal distribution was used to model the recovery value of the surface profile based on the aforementioned results.

Table 3. Properties of the Pair-Copula Families Considered.

Copula	Properties
Normal/ Gaussian (N)	Tail symmetric, no tail dependence
Student's t copula (t)	Tail-symmetric, tail dependence
Clayton (C)	Tail-asymmetric, reflection-asymmetric, suitable for modeling lower tail dependence (no upper tail dependence)
Gumbel (G)	Tail-asymmetric, reflection-asymmetric, suitable for modeling upper tail dependence (no lower tail dependence), suitable for highly correlated variables at high values and less correlated values at low levels
Joe (J)	Tail-asymmetric, suitable for modeling upper tail dependence (no lower tail dependence)
Frank (F)	Tail-symmetric, no tail dependence, tends to work well when tail dependence is very weak.
Clayton–Gumbel (BB1)	Tail-asymmetric, suitable for different nonzero upper and lower tail dependence
Joe–Clayton (BB7).	Tail-asymmetric, suitable for different nonzero upper and lower tail dependence
Rotations of Archimedean copulas	Suitable for modeling various forms of negative dependence

Table 4. Results for the Fitted Distribution to Recovery Values for SD Surface.

	Kolmogorov–Smirnov		Anderson–Darling		Chi-squared	
Distribution	Statistic	*p*-value	Statistic	*p*-value	Statistic	*p*-value
Log-normal (3P)	0.054	0.992	0.156	*	2.91	0.709
Log-normal	0.057	0.996	0.153	0.953	2.94	0.714
Weibull (3P)	0.111	0.491	4.66	0.143	N/A	N/A
Weibull	0.098	0.656	0.962	0.023	2.91	0.573
Gamma (3P)	0.092	0.730	4.39	*	N/A	N/A
Gamma	0.150	0.166	1.66	0.038	9.67	0.085
Normal	0.227	0.007	4.86	0.003	10.1	0.017
Logistic	0.229	0.006	2.91	0.030	8.42	0.038
Exponential	0.096	0.678	0.903	0.151	7.01	0.220
Exponential (2P)	0.097	0.668	0.998	0.231	1.92	0.860

*A closed form expression for p-value does not exist.

Table 5. Results for the Fitted Distribution to Values before Tamping for SD Surface.

	Kolmogorov–Smirnov		Anderson–Darling		Chi-squared	
Distribution	Statistic	*p*-value	Statistic	*p*-value	Statistic	*p*-value
Log-normal (3P)	0.086	0.797	0.345	*	5.15	0.397
Lognormal	0.148	0.175	1.17	0.282	4.69	0.320
Weibull (3P)	0.104	0.576	4.71	0.035	N/A	N/A
Weibull	0.189	0.040	3.30	0.038	10.2	0.037
Gamma (3P)	0.109	0.523	4.71	*	N/A	N/A
Gamma	0.187	0.042	2.84	0.064	15.0	0.005
Normal	0.255	0.002	5.16	0.002	12.3	0.006
Logistic	0.258	0.001	4.90	0.015	11.8	0.019
Exponential	0.264	9.4E-04	4.62	0.004	21.5	2.6E-04
Exponential (2P)	0.138	0.239	1.35	0.100	8.27	0.141

*A closed form expression for p-value does not exist.

Similarly, the three-parameter log-normal distribution was also found to have the best-fit for both the SD surface values before tamping and SD surface values after tamping as shown in Tables 5 and 6, respectively.

Copula fitting. The Gumbel copula was found to pro- vide the best-fit of the underlying dependence between the SD surface values before tamping and the recovery values. The Gumbel copula produced both the lowest AIC and BIC values as shown in Table 7.

The selection of the Gumbel copula suggests an asymmetric dependence (specifically an upper tail dependence) between the track quality (standard deviation surface) before tamping and the recovery value. Upper tail dependence means that the pair is highly correlated at high values (upper tail of the distributions) but lowly correlated at lower values.

Simulated values were generated given the three- parameter log-normal marginals (for both track quality before tamping and recovery value) and Gumbel copula. An illustrative comparison between the real and simulated values for recovery values against track condition before tamping for SD surface is shown in Figure 1.

The Joe–Clayton (popularly known as BB7) copula was found to offer best-fit of the underlying dependence between the track quality (SD surface) before tamping and the track quality after tamping. The BB7 copula was found to produce the lowest AIC and BIC values as shown in Table 8.

Table 6. Results for the Fitted Distribution to Values after Tamping for SD Surface.

	Kolmogorov–Smirnov		Anderson–Darling		Chi-squared	
Distribution	Statistic	*p*-value	Statistic	*p*-value	Statistic	*p*-value
Log-normal (3P)	0.100	0.628	0.404	*	4.84	0.436
Log-normal	0.154	0.146	1.88	0.109	4.01	0.405
Weibull (3P)	0.114	0.467	4.92	0.007	N/A	N/A
Weibull	0.186	0.044	3.81	0.021	10.2	0.037
Gamma (3P)	0.121	0.392	1.40	*	5.04	0.284
Gamma	0.210	0.016	3.03	0.032	11.3	0.024
Normal	0.244	0.003	5.16	0.002	9.73	0.021
Logistic	0.245	0.003	3.82	0.011	11.5	0.009
Exponential	0.323	2.0E-05	5.70	0.001	26.1	3.0E-05
Exponential (2P)	0.129	0.310	1.60	0.044	12.0	0.018

*A closed form expression for p-value does not exist.

Table 7. Results for the Fitted Bivariate Copula between Values before Tamping and Recovery Values for SD Surface.

Copula	Parameter 1	Parameter 2	AIC	BIC	Kendall's tau
Gumbel	$\theta = 3.31$	–	−79.95	−77.98	0.7
BB7	$\theta = 4.17$	$\delta = 1.2$	−78.95	−75.01	0.68
BB6	$\theta = 1.59$	$\delta = 2.45$	−78.53	−74.59	0.69
BB1	$\theta = 0$	$\delta = 3.31$	−77.95	−74.01	0.7
Joe	$\theta = 4.61$	–	−77.23	−75.26	0.65
BB8	$\theta = 4.61$	$\delta = 1$	−75.23	−71.29	0.65
Gaussian/Normal	$\rho = 0.89$	–	−74.50	−72.53	0.7
Student's t copula	$\rho = 0.89$	$\nu = 30$	−72.42	−68.48	0.7
Frank	$\theta = 9.98$	–	−65.65	−63.68	0.67
Clayton	$\theta = 2.35$	–	−47.97	−46.00	0.54

BIC: Bayesian information criterion; AIC: Akaike information criterion.

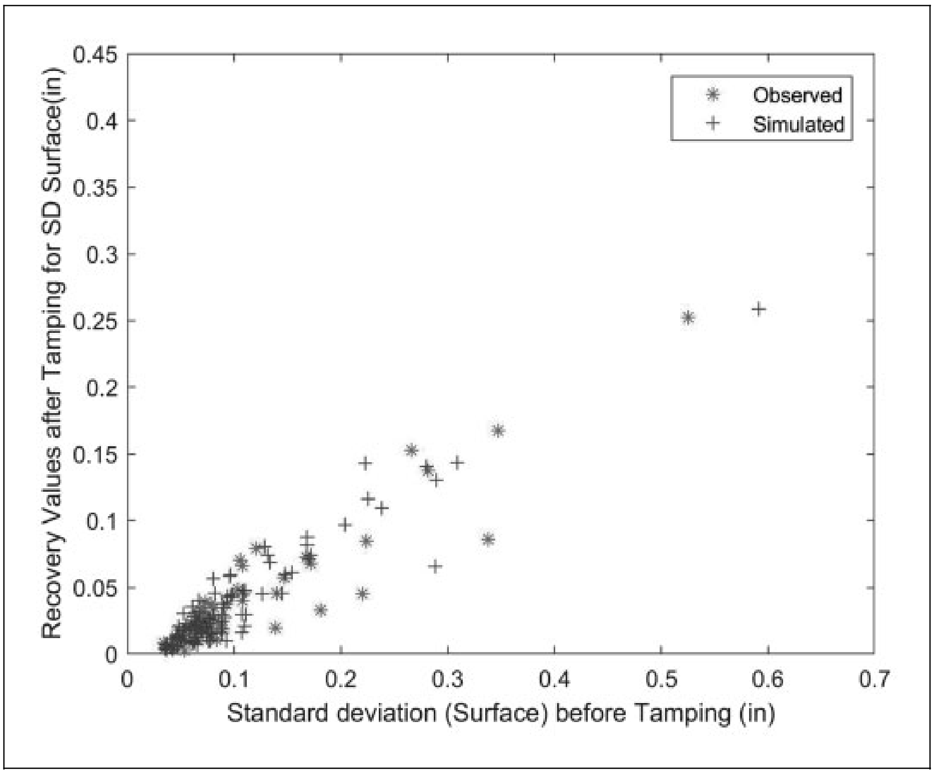

Figure 1. Comparison between real and simulated values for SD surface given the three-parameter log-normal marginals (before tamping and recovery values) and Gumbel copula.

Table 8. Results for the Fitted Bivariate Copula between Values before and after Tamping for SD Surface.

Copula	Parameter 1	Parameter 2	AIC	BIC	Kendall's tau
BB7	$\theta = 4.36$	$\delta = 2.3$	−89.26	−85.32	0.72
Gumbel	$\theta = 3.56$	–	−85.24	−83.27	0.72
BB1	$\theta = 0.18$	$\delta = 3.3$	−83.54	−79.60	0.72
BB6	$\theta = 1.32$	$\delta = 2.97$	−83.42	−79.48	0.72
Student's t copula	$\rho = 0.89$	$\nu = 2$	−82.60	−78.67	0.69
Joe	$\theta = 4.84$	–	−80.77	78.80	0.67
BB8	$\theta = 4.84$	$\delta = 1$	−78.77	−74.83	0.67
Gaussian/Normal	$\rho = 0.89$	–	−76.72	−74.75	0.7
Frank	$\theta = 10.56$	–	−66.71	−64.74	0.68
Clayton	$\theta = 2.89$	–	−58.85	−56.88	0.59

BIC: Bayesian information criterion; AIC: Akaike information criterion.

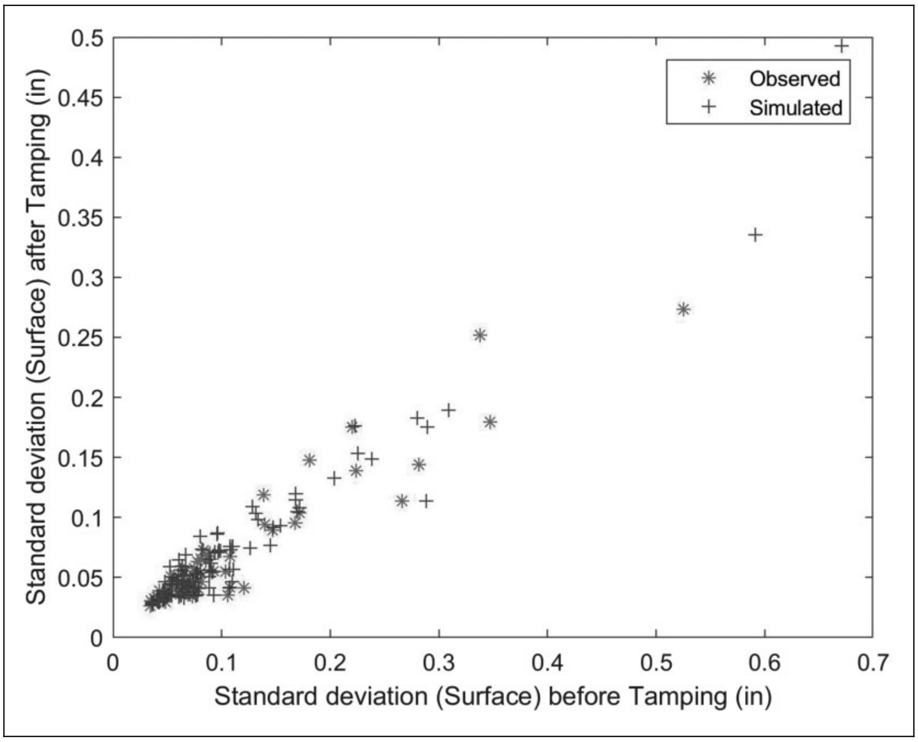

Figure 2. Comparison between real and simulated values for SD surface given the three-parameter log-normal marginals (before tamping and after tamping) and Joe–Clayton (BB7) copula.

The Joe–Clayton copula consists of the Joe copula and Clayton copula, which are suitable for modeling upper tail and lower tail dependence, respectively. The selection of the BB7 copula suggests an asymmetric dependence (with different nonzero lower and upper tail dependence coefficients) between the SD surface values before tamping and SD surface values after tamping. Similarly, simulated values were generated given the three-parameter log-normal marginals (for both track quality before tamping and track quality after tamping) and Joe–Clayton (BB7) copula. An illustrative comparison between the real and simulated values for track condition after tamping against track condition before tamping for SD surface is shown in Figure 2.

Alignment

Marginal fitting. Similar to the surface profile results, the three-parameter log-normal distribution was found to have the best-fit for the recovery values of SD alignment after tamping. The three-parameter log-normal distribution has previously been used to model the recovery values of SD alignment by Soleimanmeigouni et al. (2016b). The three-parameter log-normal distribution was also found to have the best-fit of track quality (SD alignment) values before tamping and after tamping.

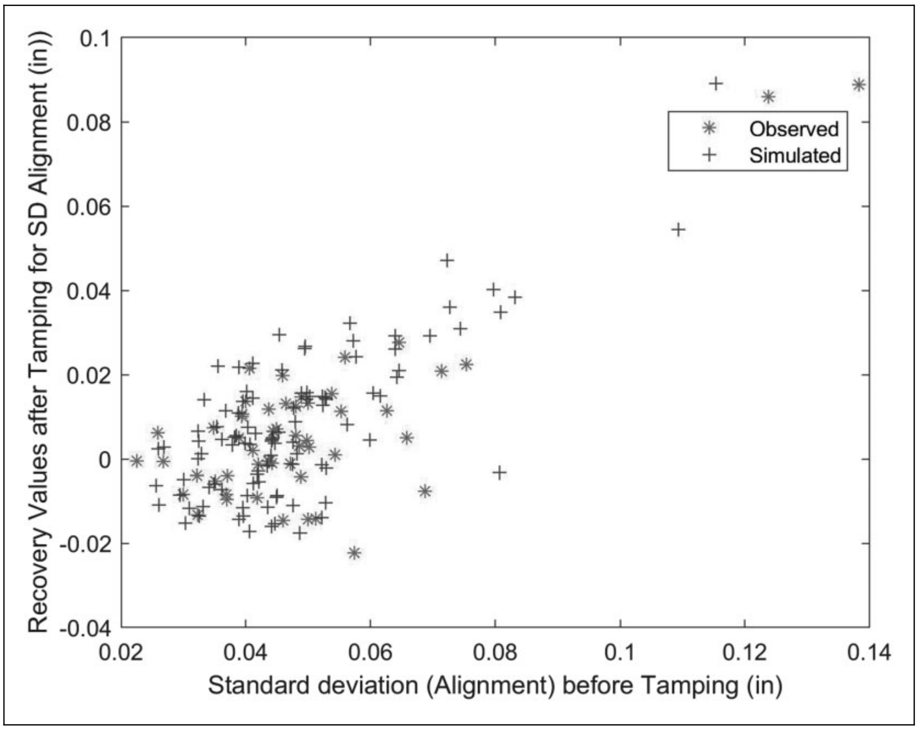

Figure 3. Comparison between real and simulated values for SD alignment given the three-parameter log-normal marginals (before tamping and recovery values) and Joe copula.

Copula fitting. The Joe copula provided the best-fit of the underlying dependence between the SD alignment values before tamping and the tamping recovery values producing both the lowest AIC and BIC values. The selection of the Joe copula suggests an upper tail dependence between the SD alignment values before tamping and tamping recovery values. The Joe copula has an even stronger positive upper tail dependence in comparison to the Gumbel copula and can be observed by tighter clustering of observations in the upper tail.[39] Simulated values were generated given the three-parameter log-normal margins (for SD alignment values before tamping and recovery values) and Joe copula. Figure 3 shows the comparison between the observed and simulated values for recovery values against SD alignment before tamping.

The Gaussian (or Normal) copula offered the best- fit of the underlying dependence between the SD alignment values before tamping and SD alignment values after tamping. The selection of the Gaussian copula suggests that the underlying dependence between the pair is radially symmetric with strong central dependence and very weak tail dependency. Similarly, simulated values were produced given the three-parameter log-normal marginals (for both SD alignment before tamping and SD alignment after tamping) and Gaussian copula. An illustrative comparison between the real and simulated values for track condition after tamping against track condition before tamping for SD alignment is shown in Figure 4.

Cross level

Marginal fitting. The three-parameter log-logistic distribution was found to best fit the recovery values of SD cross level. The three-parameter log-logistic distribution has an identical shape to the three-parameter log- normal distribution (which was found to be the next best distribution) but has heavier tails. The three- parameter log-normal distribution was also found to have the best-fit of track quality (SD cross level) values before tamping and after tamping.

Copula fitting. The bivariate asymptotic independence test performed prior to copula fitting and selection determined that the recovery values of the cross level and the track quality (SD cross level) before tamping were independent. The *p*-value of 0.43 was found to be higher than the 0.05 significance level. Thus, the null hypothesis that the variables are independent was accepted and the independence copula was selected for the pair of variables. Ignoring the test would have led to the selection of the Joe copula of parameter value of 1.38 and Kendall's tau of 0.18. Simulated values were generated given the three-parameter log-normal marginal (before tamping), three-parameter log-logistic marginal (recovery values), and independence copula. The simulated values are illustrated in Figure 5.

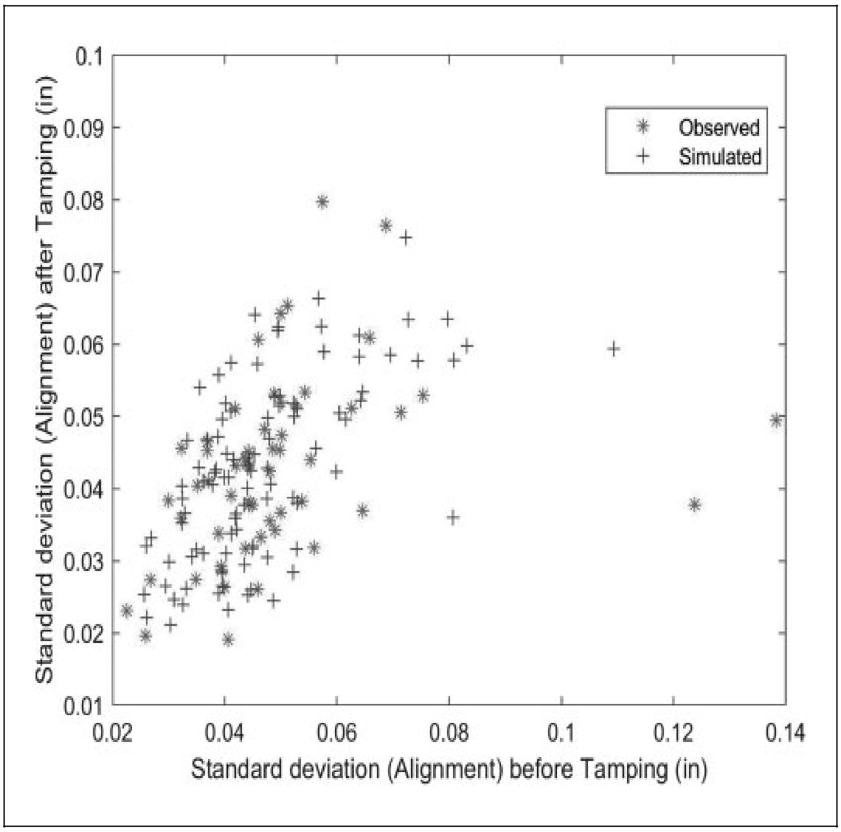

Figure 4. Comparison between real and simulated values for SD alignment given the three-parameter log-normal marginals (before tamping and after tamping) and Gaussian (normal) copula.

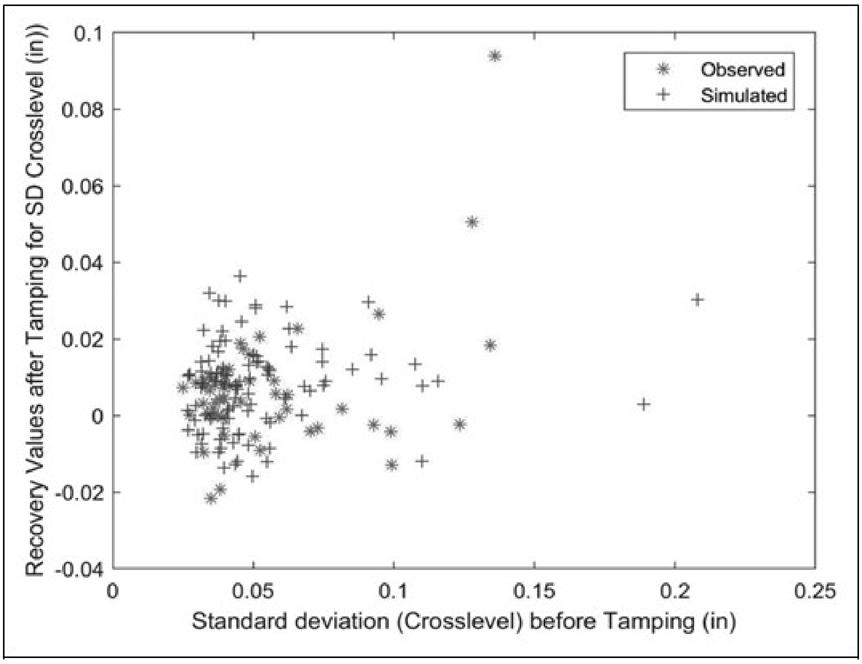

Figure 5. Comparison between real and simulated values for SD cross level given the three-parameter log-normal marginal (before tamping), three-parameter log-logistic marginal (recovery values), and independent copula.

Warp

Marginal fitting. Similar to the cross-level results, the three-parameter log-logistic distribution was found to have the best-fit for the tamping recovery values of SD warp. Furthermore, the three-parameter log- normal distribution was also found to have the best-fit of track quality (SD warp) values before tamping and after tamping.

Copula fitting. The Joe copula provided the best-fit of the underlying dependence between the SD warp values before tamping and the tamping recovery values. The Joe copula produced both the lowest AIC and BIC values. Simulated values were generated given the three-parameter log-normal marginal (before tamping), three-parameter log-logistic marginal (recovery values), and Joe copula. The simulated values are illustrated in Figure 6.

Gage

Marginal fitting. The three-parameter log-logistic distribution was found to have the best-fit for the tamping recovery values of SD gage. The three-parameter log- logistic distribution offered the lowest statistic for all three tests. It also had a p-value far greater than 0.05 for the KS and Chi-squared tests meaning the null hypothesis that it follows the distribution can be accepted. The closed-form expression for the p-value of the three-parameter log-logistic distribution, how- ever, does not exist for the Anderson–Darling test. The two-parameter log-normal distribution and the three-parameter log-logistic distribution were found to provide the best fit for both SD gage values before tamping and SD gage values after tamping, respectively.

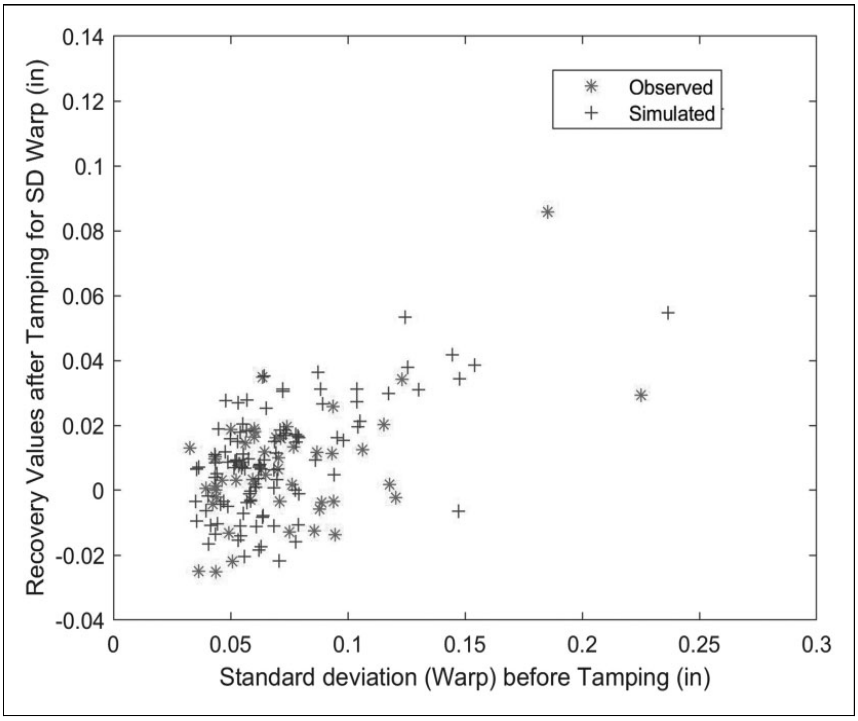

Figure 6. Comparison between real and simulated values for SD warp given the three-parameter log-normal marginal (before tamping), three-parameter log-logistic marginal (recovery values), and Joe copula.

Copula fitting. The Joe–Frank (BB8) copula was found to offer the best-fit of the underlying dependence between the SD gage values before tamping and tamping recovery values producing both the lowest AIC and BIC values. The BB8 copula consists of the Joe copula and Frank copula. The Joe copula is suitable for strong upper tail dependence, whereas Frank copula is suitable for very strong central dependence with very weak tail dependence. The Frank copula has stronger central dependence than the Gaussian copula (denoted by significant central clustering) and even weaker tail dependence than the Gaussian copula (denoted by fanning out at the tails) (Bhat and Eluru 2009). Simulated values were generated given the two-parameter log-normal (values before tamping), three-parameter log-logistic distribution (recovery values), and Joe–Frank (BB8) copula. The comparison of the observed and simulated values is shown in Figure 7.

Correlation analysis of recovery values of geometric parameters

Correlation analysis was conducted to measure the dependence between the tamping recoveries of the various track geometric parameters namely surface profile, alignment, cross level, warp, and gage. The correlation measures employed include Pearson's correlation coefficient and concordance (or rank correlation) measures such as Kendall's tau and Spearman's rho. Pearson's correlation coefficient measures the linear dependence between random variables and assumes that the variables of interest are normal. Thus, the widely used Pearson's coefficient is not suitable for evaluating the nonlinear dependence or dependence between non- normal distributions.

The results of the linear correlation analysis are shown in Table 9. The highest dependence was found between the recoveries of SD warp and SD cross level. The fact that warp is a measure of the cross-level variation offers some support to the high dependence observed. On the other hand, the lowest dependence was observed between the recovery values of SD gage and SD surface. Gage is a trans- verse horizontal parameter, whereas surface is a vertical longitudinal parameter. In fact, generally gage was found to have relatively weak dependences between the other parameters with its highest

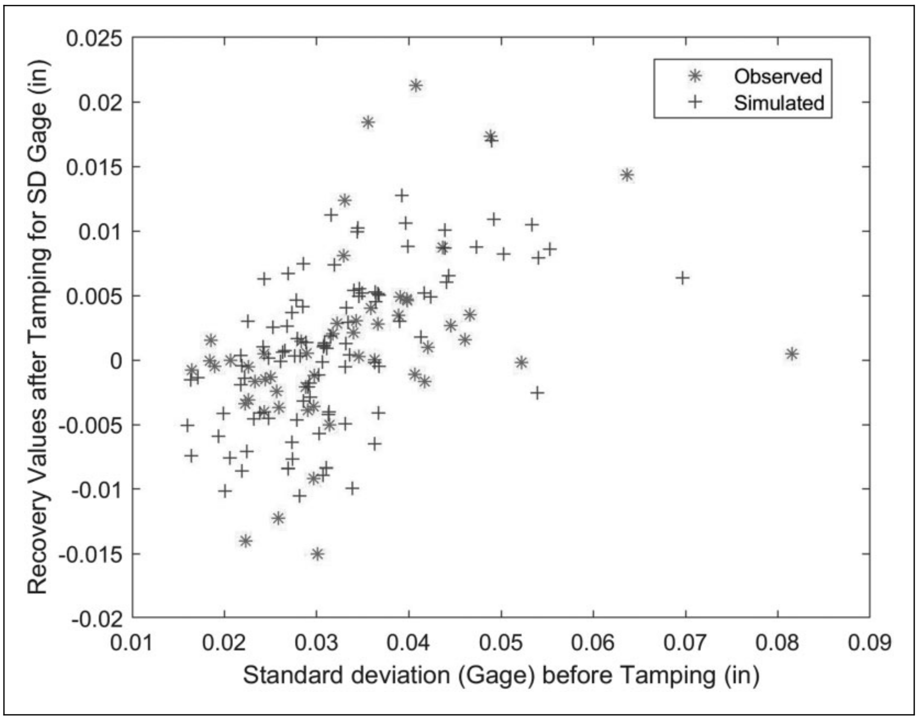

Figure 7. Comparison between real and simulated values for SD gage given the two-parameter lognormal marginal (before tamping), three-parameter log-logistic marginal (recovery value), and Joe–Frank (BB8) copula.

Table 9. Pearson's Correlation Matrix of Recovery Values of the Geometric Parameters.

Parameter	Surface	Alignment	Cross level	Gage	Warp
Surface	1.00	0.50	0.30	−0.02	0.37
Alignment	0.50	1.00	0.53	0.19	0.61
Cross level	0.30	0.53	1.00	0.07	0.80
Gage	−0.02	0.19	0.07	1.00	0.13
Warp	0.37	0.61	0.79	0.13	1.00

Table 10. Kendall's tau Correlation Matrix of Recovery Values of the Geometric Parameters.

Parameters	Surface	Alignment	Cross level	Gage	Warp
Surface	1.00	0.17	0.07	0.02	0.15
Alignment	0.13	1.00	0.06	0.24	0.31
Cross level	0.07	0.06	1.0	0.12	0.35
Gage	0.02	0.24	0.12	1.00	0.12
Warp	0.15	0.31	0.35	0.11	1.00

dependence observed with alignment, which is a horizontal longitudinal parameter unlike the others. The surface profile was found to have moderate dependence with both cross level and warp, which are also vertical parameters. Alignment was found to have moderate correlations with vertical parameters such as surface, cross level, and warp parameters. Of these three parameters, surface profile was the parameter with the highest dependence with alignment, which suggests that tamping affects the surface profile in more similar way to alignment in comparison with the others. Surface and alignment are both longitudinal parameters.

However, a comparison of the results of linear correlation with the results of concordance dependence shows a general reduction in the observed dependence between the recoveries of the various parameters. These are shown in the results of concordance dependence such as the Kendall's tau and Spearman's rho correlation matrices in Tables 10 and 11, respectively. For instance, the linear dependence of 0.80 was found to reduce to 0.35 and 0.47 by employing the Kendall's tau and Spearman's rho dependence measures, which do not assume linear dependence or assume normality of the random variables. As a matter of fact, the recoveries of warp and cross level and warp were found to assume three-parameter log-normal distribution and three-parameter log-logistic distribution. Furthermore, the normal distribution was found to not fit the data as shown in the aforementioned tables. Additionally, an examination of the under- lying dependence suggests a Student's t copula. Thus, it may be quite misleading to employ linear correlation coefficient not only in modeling the tamping recovery of various parameters but also in analyzing the dependences of the various recoveries of these parameters.

Table 11. Spearman's rho Correlation Matrix of Recovery Values of the Geometric Parameters.

Parameters	Surface	Alignment	Cross level	Gage	Warp
Surface	1.00	0.18	0.08	0.04	0.22
Alignment	0.18	1.00	0.08	0.35	0.46
Cross level	0.08	0.08	1.00	0.16	0.47
Gage	0.04	0.35	0.16	1.00	0.17
Warp	0.21	0.46	0.47	0.17	1.00

CONCLUDING REMARKS

The effect of tamping on various parameters namely surface, alignment, cross level, warp, and gage were evaluated by analyzing the recovery of these geometric parameters after tamping. Tamping recovery has been found to be predominantly dependent on the track geometry condition before tamping. It has largely been modeled using deterministic techniques such as linear regression, which assumes multivariate normal distribution and linear relationship between the variables. However, non-normality in most cases transpires in various forms: non-normality of marginal distribution of some variables and in some instances multivariate non-normality of the joint distribution of a group of variables despite normal marginal distributions of all the individual variables. Furthermore, deterministic techniques are not suitable given the high degrees of uncertainty that happens to be observed in the recovery values of track geometry measures in majority of cases. Thus, probabilistic techniques are increasingly being employed that take into consideration the high variation in the restoration values after tamping even for similar track geometry condition. Majority of studies do not take into consideration the underlying dependence between the variables of interest. Thus, the authors employ a copula-based approach to model the tamping recovery phenomenon by combining arbitrary marginal distributions to form a joint distribution with the underlying dependence.

From marginal fitting results, the recoveries of the various parameters were found to be non-normal and were found to either fit a three-parameter log-normal distribution (in the case of surface, alignment, and warp) or three-parameter log-logistic distribution (in the case of cross level and gage). Similarly, non- normal distributions were observed for the track quality condition (SD of track geometric parameters) before and after tamping. Various copulas were fitted in order to find the copula, which best describe the underlying dependence between the variables. The selection of copulas such as Gumbel, Joe, and Joe–Clayton copulas (BB7) suggest the presence of asymmetric and tail dependence, which cannot be appropriately captured using the widely used linear regression. Thus, conventional correlation analysis appears not to be suitable for analyzing the dependences between the recovery values and tamping condition before tamping.

Correlation analysis of the recovery of various geometric parameters shows that the use of Pearson's correlation coefficient, which assumes normality of the variables and linear dependence, led to the observation of relatively high dependence values. However, the use of concordance measures such as Kendall's tau and Spearman's rho resulted in a general reduction in the observed dependences. These concordance measures are scale-invariant and are suitable for evaluating nonlinear dependence and measure dependence irrespective of the assumed distribution. Thus, the widely used Pearson's correlation coefficient does not appear to be appropriate for analyzing the correlation between the recoveries of the various track geometric parameters. From the correlation analysis results, the strongest correlation was observed between warp and cross-level recoveries with the weakest correlation observed between the surface and gage recoveries with varying levels in-between. This infers and gives credence to the previous research that tamping affects the various track geometric parameters differently and, thus, it is imperative to examine all the track geometric parameters and not focus on one or two parameters.

The copula-based approach was employed by considering only the predominant factor, which is the track geometry condition or quality before tamping. However, this methodology can be extended to incorporate and examine other factors such as operational speed, tamping procedure, age of track components, and number of previous tamping operations. In order to analyze the dependences between more than two variables, vine copulas are suggested, which are more flexible than regular multivariate copulas. Vine copulas employ arbitrary bivariate copulas as building blocks for the construction of higher dimensional multivariate distributions.

The copula-based tamping recovery model can be incorporated into track geometry maintenance scheduling models with the track geometry degradation models and recovery models being the main components of these models. Degradation models that can be considered include linear and exponential regression models, polynomial models, multi-stage linear models, neural networks, grey models, path analysis, data mining, models with random coefficient, Markov models, time series models, and stochastic processes. There is the need to select an appropriate track geometry deterioration model that takes into consideration both the time and spatial variation of the track geometry degradation process.[9] The combination of such a model with a copula-based approach that models the tamping recovery phenomena considering the underlying dependence will lead to better track geometry condition estimation for the planning of maintenance activity. The combination of such models will also result in a greater comprehension of track geometry maintenance modeling. This proposed methodology will be considered in a future case study. In order to integrate such degradation models and copula-based recovery models in track scheduling models, probabilistic optimization models need to be considered.

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