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Invited Talks

Reflections on Colorings

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We look back at the topic of graph colorings and describe some related concepts. Some results and problems concerning these coloring concepts are presented.

Iterated Partitions of Triangles

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There are many ways in which one can subdivide a triangle into smaller triangles. However, the limiting behavior when various methods of partitioning are iterated can be quite different. In this talk, I will describe some recent results for this problem, which include a few facts that we can prove, and a large set of conjectures arising from computational experiments that we cannot (yet) prove. This is joint work with Steven Butler

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Resolutions of t -designs were studied as early as 1847 by Reverend T. P. Kirkman who proposed the famous 15 schoolgirls problem. Kirkman's problem is equivalent to finding a *resolvable* 2 -($15, 3, 1$) design with $r = 7$, and $b = 35$. We define and discuss τ -resolutions of t -designs, *large sets* of t -designs, *orthogonal resolutions* of t -designs, *Room rectangles*, and *Steiner tableaux*. In particular we discuss recursive constructions of large sets, and present techniques for constructing *starter* large sets of t -designs, which combine with the recursive techniques to yield infinite families of large sets. We pay particular attention to *coherence* techniques, i.e. construction methods which assume particular automorphism groups under which the objects mentioned above are invariant. Examples of large sets and *super-large* sets constructed by these methods are presented. We present infinite families of *semiregular* large sets arising from 3-homogeneous actions of the groups $PSL_2(q)$ on the projective line, and provide a generalization of these results to the case where the group action is not t -homogeneous. We finally present some tantalizing open problems.

Mix Functions and Orthogonal Equitable Rectangles

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Ristenpart and Rogaway defined “mix” functions, which are used to mix inputs from two sets of equal size and produce outputs from the same two sets, in an optimal way. These functions have a cryptographic application in the context of extending the domain of a block cipher. It was observed that mix functions could be constructed from orthogonal latin squares.

We give a simple, scalable construction for mix functions. We also consider a generalization of mix functions, in which the two sets need not be of equal size. These generalized mix functions turn out to be equivalent to an interesting type of combinatorial design which has not previously been studied. We term these “orthogonal equitable rectangles” and we construct them for all possible parameter situations.

The Symmetry Between Crossings and Nestings in Combinatorics

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In this talk we survey the recent progress on crossings and nestings in combinatorial structures. For permutations, crossings and nestings are just the increasing and decreasing subsequences, which bear a clear symmetry. Such a symmetry also holds for matchings and set partitions. By a variant of the RSK algorithm from algebraic combinatorics, matchings and set partitions are in one-to-one correspondence with certain random walks in Young lattice—the lattice of integer partitions. It follows that the crossing numbers and the nesting numbers are distributed symmetrically over all matchings of $[2n]$, as well as all partitions of $[n]$.

A similar technique is used to reveal the symmetry between monotone substructures of other geometric configurations, including fillings of Ferrers shapes, linked cycles, general graphs, and fillings of stack and moon polyominoes.

Contributed Talks

Statistical Behavior of Perturbed Logistic Model

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A model represents the rate of changes of the population with a limited environmental resources can be described by $\frac{dP}{dt} = P(a - bP) + g(t, P)$, $P(t_0) = P_0$, where a measures the growth rate in the absence of the restriction force and a/b represents the carrying capacity of the environment and b represent restricted factor b . The random perturbation $g(t, P)$ is generated by random change in the environment. The behavior of the solution of this model for continuous and discrete case when $g(t, P) = r.P$ with a random change factor r will be studied. The stability and the behavior of the equilibrium point will also be investigated. A computational approach to the solution and logistic regression applied to the statistical data will be presented.

Generalized Complementary Prisms and Their Parametric Spread

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For a graph G with complement \bar{G} and a bijection $\pi : V(G) \rightarrow V(\bar{G})$, the complementary prism $G\pi\bar{G}$ is the graph obtained by taking disjointed copies of G and \bar{G} and adding the edge $\{v, \pi(v)\}$ for each $v \in V(G)$. For a given graph parameter τ , we define the complementary spread $\overline{SP}_\tau(G)$ to be the maximum value of $\tau(G\pi_1\bar{G}) - \tau(G\pi_2\bar{G})$ for bijections $\pi_i : V(G) \rightarrow V(\bar{G})$, $i = 1, 2$. We focus primarily on the independence parameter $\beta(G)$.

Extending Partial Tournaments

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Let A be a $(0; 1; *)$ -matrix with main diagonal all 0's and such that if $a_{ij} = 1$ or $*$, then $a_{ji} = *$ or 0. Under what conditions on the row sums, and or column sums, of A is it possible to change the $*$'s to 0's or 1's and obtain a tournament matrix (the adjacency matrix of a tournament digraph) with a specified score sequence? We answer this question in the case of regular and nearly regular tournaments. The result we give is best possible in the sense that no relaxation of any condition will always yield a matrix that can be so extended.

On Nonlinear Extensions of the Erdős-Ginzburg-Ziv Theorem

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Let p be a prime. We consider a homogenous system $\Phi = \{\varphi_i\}_{i=1}^k$ of k symmetric polynomials in p variables over the field \mathbb{Z}_p in view of the EGZ theorem. Denote by $g(\Phi, \mathbb{Z}_p)$ the minimum integer s which satisfies the following property: For every sequence of s elements a_1, a_2, \dots, a_s from \mathbb{Z}_p , there exist p distinct integers $i_1, i_2, \dots, i_p \in \{1, 2, \dots, s\}$ such that $\varphi_i(a_{i_1}, a_{i_2}, \dots, a_{i_p}) = 0$ for $i = 1, 2, \dots, k$. Furthermore, denote by $M(\Phi, \mathbb{Z}_p)$ the set of all sequences of elements from \mathbb{Z}_p of length $g(\Phi, \mathbb{Z}_p) - 1$ which do not satisfy the above property.

The Erdős-Ginzburg-Ziv theorem states that if $\Phi = \{s_1\}$, where $s_1 = \sum_{i=1}^p x_i$, then $g(\Phi, \mathbb{Z}_p) = 2p - 1$, furthermore, the set $M(\Phi, \mathbb{Z}_p)$ is the set of all two element sequences taken from \mathbb{Z}_p , where each element appears $p - 1$ times. Motivated by this theorem which generalizes the pigeon-hole principle for $2p - 1$ pigeons and two holes, we propose to investigate systems of symmetric polynomials which generalize the pigeon-hole principle for $t(p - 1) + 1$ pigeons and t holes. It is worthwhile to mention that while the system Φ has to satisfy $g(\Phi, \mathbb{Z}_p) = t(p - 1) + 1$, there is some flexibility in defining the set $M(\Phi, \mathbb{Z}_p)$.

In this talk we will survey some recent results and suggest further avenues of research.

New Results on Packings

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A 2 - (v, k, λ) packing design, $(\mathcal{V}, \mathcal{B})$, is a set \mathcal{V} (with elements called **points**) and a collection \mathcal{B} of k -subsets of \mathcal{V} (called **blocks**) with the property that every unordered pair of points occurs in at most λ blocks. We denote the maximum possible size of \mathcal{B} by $D_\lambda(v, k, 2)$ and call it the **packing number** for these parameters. We are interested in finding either the exact value of $D_\lambda(v, k, 2)$ or a good lower bound on it. I will discuss several computational results on improving the known bounds on the size of constant weight codes (packings with $\lambda = 1$) and an update on the exact values of $D_\lambda(v, 5, 2)$. Most of the results were found by using combinatorial optimization.

Progressions of Squares

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We consider the occurrence of certain generalizations of arithmetic progressions, amongst the squares of integers. More specifically, let a_1, a_2, \dots, a_n be an increasing sequence of squares, let $d_i = a_{i+1} - a_i$, and let $D = \{d_1, d_2, \dots, d_{n-1}\}$. If the given sequence of squares is an arithmetic progression (impossible if $n > 3$) then D has size 1 and diameter 0. When $n > 3$, we seek to minimize either the size of D or the diameter of D .

On Some Projective Planes

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In this study we analyze the structure of the full collineation group of the Veblen-Wedderburn(VW) plane of order 11^2 and Hughes plane of order 11^2 . Particularly, we discuss a new construction method of VW_{121} by using its collineation group, and it seems that this method could be used for VW planes of other orders as well. Furthermore, we analyze the subplane structures of these planes and determine the orbit representatives of all proper subplanes generated by the quadrangles under their full collineation groups.

Beautifully Nested Balanced Incomplete Block Designs

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We define Beautifully Nested Balanced Incomplete Block Designs and prove that the necessary conditions are sufficient for the existence of BNBIBD with block size 3. Several infinite families with block size 4 are constructed and some non-existence results along with two well supported conjectures are also given.

A construction of some group divisible designs

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In this talk, we present a construction of two families of group divisible designs with block sizes 3 and 4, respectively. It is motivated by the decoding of quadratic residue codes.

Some New Results on the Existence of Balanced Arrays

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A balanced array T (B-array) of strength t , with m constraints (factors), and N runs (treatment-combinations) is merely a matrix of size $(m \times N)$ such that if we choose any submatrix T^* of T with t ($t \leq m$) rows, then the structure of T^* is such that every $(t \times 1)$ column vector of weight i ($0 \leq i \leq t$; the weight of a vector is the number of 1s in it) appears with the same frequency μ_i (say; $0 \leq i \leq t$). The vector $\underline{\mu}' = (\mu_0, \mu_1, \dots, \mu_t)$ is called the index set of the array. Clearly, N is known once we know the μ_i 's. Arrays with combinatorial structures have proven to be quite useful in combinatorics and statistical design of experiments. In this paper, we consider the existence of some such arrays by using the concept of moments which are extensively used in statistics.

On \mathbb{Z}_3 -magic Graphs

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For $k > 0$, we call a graph $G = (V, E)$ as \mathbb{Z}_k -magic if there exists a labeling $l : E(G) \rightarrow \mathbb{Z}_k$ such that the induced vertex set labeling $l^+ : V(G) \rightarrow \mathbb{Z}_k$ defined by $l^+(v) = \sum \{l(u, v) : (u, v) \in E(G)\}$ is a constant map. We denote the set of all k such that G is k -magic by $IM(G)$. We call this set as the integer-magic spectrum of G . We investigate graphs which are \mathbb{Z}_3 -magic and non \mathbb{Z}_3 -magic.

On the Edge-Balance Index Sets of Envelope Graph of Stars, Paths and Cycles

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Let G be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $\mathbb{Z}_2 = \{0, 1\}$. A labeling f of a graph G is said to be edge-friendly if $|e_f(0) - e_f(1)| \leq 1$. An edge-friendly labeling $f : E(G) \rightarrow \mathbb{Z}_2$ induces a partial vertex labeling $f^+ : V(G) \rightarrow \mathbb{Z}_2$ defined by $f^+(x) = 0$ if the number of edges incident on x is 0 more than 1. Similarly, $f^+(x) = 1$ if the number of edges incident on x is 1 more than 0. $f^+(x)$ is not define if the number of edges incident on x is 1 equal to the number of edges labeled by 0. For $i \in \mathbb{Z}_2$, let $v_f(i) = \text{card}\{v \in V(G) : f^+(v) = i\}$ and $e_f(i) = \text{card}\{e \in E(G) : f(e) = i\}$. The edge-balance index set of the graph G , $\text{EBI}(G)$, is defined as $\{|v_f(0) - v_f(1)| : \text{the edge labeling } f \text{ is edge-friendly}\}$. Given a graph G , the envelope graph $EV(G)$ is the graph with $V(EV(G)) = V(G) \cup E(G)$ and $E(EV(G)) = E(G) \cup \{(u, (u, v)) : u \in V, (u, v) \in E(G)\}$. The edge-balance index sets of envelope graphs of stars, paths and cycles are presented.

Fair Difference Systems of Sets and the Comma-Free Index

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Finite difference systems of sets (DSS) were introduced by Levenshtein in 1971 to determine systematic comma-free codes with minimal redundancy in the presence of errors. The case for two sets had been studied previously by D. J. Clague. If the sets differ in size by at most one element the system is called fair.

The index of a comma-free code is the minimal distance between codewords and overlaps of code words. We determine conditions on fair DSS that imply the index of the corresponding comma-free code is greater than one.

Some Remarks on Matrix Enumeration

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In March 2008, Neil J. Calkin and Janine E. Janoski presented their paper "Matrices of Row and Column Sum 2" at *CGTC39*, and this paper was accepted by *Cong. Numer.* recently. They let $M(n, s)$ be the number of $n \times n$ matrices with binary entries, with rows in lexicographical order, and row and column sum s ; and $S(n, s)$ be the number of $n \times n$ matrices with entries from $\{0, 1, 2\}$, symmetric, with trace 0, and row and column sum s . The formula for $S(n, 2)$ was known. They showed that $M(n, 2) = S(n, 2)$. They also posted a conjecture: There is a bijection between these two types of matrices. We will present a bijection between them, including new proofs for $M(n, 2)$ and $S(n, 2)$, as well as other results on matrix enumeration.

Counting Integer Matrices with Small Row Sum and Column Sum

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The enumeration of Integer Matrices has been the subject of considerable study, and it is unlikely that a simple formula exists. The number in question can be related in various ways to the representation theory of the symmetric group or of the complex general linear group, but this does not make its computation any easier.

From Stanley's book (*Enumerative Combinatorics I*): "Let $f(n)$ be the number of $n \times n$ matrices M of zeros and ones such that every row and column of M has exactly three ones, $f(0) = 1$, $f(1) = f(2) = 0$, $f(3) = 1$. The most explicit formula known at present for $f(n)$ is

$$(2) \quad f(n) = 6^{-n} \sum \frac{(-1)^\beta n!^2 (\beta + 3\gamma)! 2^\alpha 3^\beta}{\alpha! \beta! \gamma!^2 6^\gamma}$$

where the sum is over all $(n+2)(n+1)/2$ solutions to $\alpha + \beta + \gamma = n$ in nonnegative integers. This formula gives very little insight into the behavior of $f(n)$, but it does allow one to compute $f(n)$ faster than if only the combinatorial definition of $f(n)$ were used. Hence with some reluctance we accept (2) as a "determination" of $f(n)$. Of course if someone were later to prove $f(n) = (n-1)(n-2)/2$ (rather unlikely), then our enthusiasm for (2) would be considerably diminished."

We will give a historical view of the related problems and present many formulas for Counting Integer Matrices with Small Row Sum and Column Sum.

Complementary to Yannakakis' Theorem

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In 1988, M. Yannakakis proved that the TSP polytope cannot be expressed by symmetric polynomial size linear program, where the symmetry means that the polytope is an invariant under vertex relabeling. The question about the size of asymmetric linear programs was left open then, and it remained open since. This study answers the question: the ATSP polytope can be expressed by asymmetric polynomial size linear program.

On Sarvate-Beam Designs

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A *Sarvate-Beam design* (V, B) is a set V of v elements and a collection B of subsets (called *blocks*) of V such that each distinct pair of elements in V occurs in a different number of blocks (in particular, exactly once from the list $1, 2, \dots, \binom{v}{2}$). In this talk, we present some results concerning SB triple systems, SB quad systems and strict aPBD designs.

Balanced Ternary Designs with Common Reductions

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In this paper, we consider balanced ternary designs, BTDs, in which every block in the design contains the same number of elements that appear doubly. We consider the two extremes, that is designs, that we call packed, where the blocks each contain as many doubletons as possible and designs, that we call monadic, where the blocks each contain a single doubleton. We determine for $k = 4$ and $k = 5$ that one can be constructed in terms of the other iff they are each nested and contain the same triple system.

Discrete Logarithms in non-Abelian Groups

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The intractability of the traditional discrete logarithm problem (DLP) forms the basis for the design of numerous cryptographic primitives. M. Sramka et al. have generalized the DLP to arbitrary finite groups. One of the reasons mentioned for this generalization is P. Shor's quantum algorithm which solves efficiently the traditional DLP. The DLP for a non-abelian group is based on a particular representation of the group and a choice of generators.

In this talk we show that care must be taken to insure that the representation and generators indeed yield an intractable DLP. We show that in $PSL(2, p) = \langle \alpha, \beta \rangle$ the discrete logarithm problem with respect to (α, β) is easy to solve for a specific representation and choice of generators α and β . As a consequence, such representation of $PSL(2, p)$ and generators should not be used to design cryptographic primitives.

We also comment on secure choices in the context of $PSL(2, p)$.

On the Edge-balance Index Sets of J-Ladders and Twisted Cylinder Graphs

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Let G be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $\mathbb{Z}_2 = \{0, 1\}$. A labeling $f : E(G) \rightarrow \mathbb{Z}_2$ of a graph G is said to be edge-friendly if $\{|e_f(0) - e_f(1)| \leq 1\}$. An edge-friendly labeling f induces a partial vertex labeling $f^+ : V(G) \rightarrow \mathbb{Z}_2$ defined by $f^+(x) = 0$ if the number of edges labeled by 0 incident on x is more than the number of edges labeled by 1 incident on x . Similarly, $f^+(x) = 1$ if the number of edges labeled by 1 incident on x is more than the number of edges labeled by 0 incident on x . $f^+(x)$ is not define if the number of edges labeled by 1 incident on x is equal to the number of edges labeled by 0 incident on x . For $i \in \mathbb{Z}_2$, let $v_f(i) = \text{card}\{v \in V(G) : f^+(v) = i\}$ and $e_f(i) = \text{card}\{e \in E(G) : f(e) = i\}$. The edge-balance index set of the graph G , $\text{EBI}(G)$, is defined as $\{|v_f(0) - v_f(1)| : \text{the edge labeling } f \text{ is edge-friendly.}\}$. In this paper, some results on the edge-balance index sets of J-ladders and twisted cylinder graphs are presented.

On Edge-Balance Index Sets of Flower Graphs

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Let $G = (V, E)$ be a simple graph, and let $A = \{0, 1\}$. Any edge labeling $f : E \rightarrow A$ induces a partial vertex labeling $f^* : V \rightarrow A$ that assigns 0 or 1 to $f^*(v)$, depending on whether there are more 0- or 1-edges incident to v , and leaves $f^*(v)$ unlabeled otherwise. For each $i \in A$, let $e_f(i) = |\{uv \in E : f(uv) = i\}|$ and $v_f(i) = |\{v \in V : f^*(v) = i\}|$. The edge-balance index set of G is defined as $\text{EBI}(G) = \{|v_f(0) - v_f(1)| : \text{the edge labeling } f \text{ satisfies } |e_f(0) - e_f(1)| \leq 1\}$. In this paper, exact values of the edge-balance index sets of flower graphs are obtained, all of them form arithmetic progressions.

On $Q(a, k)$ -Vertex-graceful Graphs

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For any integer $a, k > 1$, a graph G with vertex set $V(G)$ and edge set $E(G)$, $p = |V(G)|$ and $q = |E(G)|$, is said to be $Q(a, k)$ -vertex-graceful (in short $Q(a, k)$ -VG) if there exists a function pair (f, f^+) which assigns integer labels to the vertices and edges, i.e., $f : V(G) \rightarrow \{0, 1, \dots, p-1\}$ and $f^+ : E(G) \rightarrow \{a, a+k, a+2k, \dots, a+(q-1)k\}$ are onto, where $f^+(u, v) = f(u) + f(v)$ for any $(u, v) \in E(G)$.

We determine here classes of graphs that are $Q(a, k)$ -vertex-graceful for distinct a and k . Moreover, some conjectures are proposed.

Cryptoalgorithm Based on Formulas of Reconstruction and Decomposition on the Non-uniform Grid

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In this talk a new symmetric cryptoalgorithm based on wavelet decomposition of B-splines of the second degree on a non-uniform grid will be presented. The algorithm is simple in practical application and analysis, process of enciphering and deciphering have easy mathematical structures. Also, analysis of providing perfect secrecy of presented algorithm will be discussed.

On the Edge-balance Index Sets of Some Trees

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Let G be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $\mathbb{Z}_2 = \{0, 1\}$. A labeling $f : E(G) \rightarrow \mathbb{Z}_2$ of a graph G is said to be edge-friendly if $\{|e_f(0) - e_f(1)| \leq 1\}$. An edge-friendly labeling f induces a partial vertex labeling $f^+ : V(G) \rightarrow \mathbb{Z}_2$ defined by $f^+(x) = 0$ if the number of edges labeled by 0 incident on x is more than the number of edges labeled by 1 incident on x . Similarly, $f^+(x) = 1$ if the number of edges labeled by 1 incident on x is more than the number of edges labeled by 0 incident on x . $f^+(x)$ is not define if the number of edges labeled by 1 incident on x is equal to the number of edges labeled by 0 incident on x . For $i \in \mathbb{Z}_2$, let $v_f(i) = \text{card}\{v \in V(G) : f^+(v) = i\}$ and $e_f(i) = \text{card}\{e \in E(G) : f(e) = i\}$. The edge-balance index set of the graph G , $\text{EBI}(G)$, is defined as $\{|v_f(0) - v_f(1)| : \text{the edge labeling } f \text{ is edge-friendly.}\}$. Results parallel to the concept of friendly index sets of some trees are presented.

On the Integer-magic Spectra of Chain-sum Graphs

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Let A be a non-trivial finite abelian group and $A^* = A - \{0\}$. A graph is A -**magic** if it has an edge-labeling by elements of A^* which induces a constant vertex labeling of the graph. The **integer-magic spectrum** of a graph G is the set $\text{IM}(G) = \{k : G \text{ is } \mathbb{Z}_k\text{-magic and } k \geq 2\}$. We analyze the integer-magic spectra of various classes of chain-sum graphs.

On Splitting Strongly Regular Graphs of Latin Square Type

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Each strongly regular graph (X, R) gives rise to a two-class symmetric association scheme $(X, \{R_0, R, \bar{R}\})$, where \bar{R} is the complement of R . Now split R into a pair R_1 and R_1^T , where $R_1^T = \{(x, y) | (y, x) \in R\}$. If $(X, \{R_0, R_1, \bar{R}, R_1^T\})$ is also an association scheme, then it is called a (non-symmetric) fission scheme. We can split \bar{R} or both R and \bar{R} . In this talk, we focus on strongly regular graphs of Latin square type or negative square type which have a fission of class 3 or 4. Examples of such strongly regular graphs are not abundant.

Magic Vertex Labelings of Digraphs

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A digraph D with e edges and v vertices has a magic vertex labeling if there exists an injective map $\lambda : E(D) \rightarrow \{1, 2, \dots, v + e\}$ such that for any given vertex $x \in V(D)$ the sum of the labels going into x are equal to the sum of the labels going out of x ; however, this sum does not have to be the same for each vertex. This talk will look at some examples of magic vertex labelings, their relationship to magic digraphs, and pose some questions for future exploration.

Graceful Forests

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A graph on n vertices and m edges is *graceful* if there is a labelling of its vertices with distinct labels selected from the set $0, 1, 2, \dots, m$ such that when each edge is labelled by the absolute value of the differences of its endpoints, the edge labels are distinct. A famous open conjecture is that all trees are graceful. By this definition, a forest is not graceful unless it is a tree because with less than $n - 1$ edges, the vertices cannot be assigned distinct labels using the integers 0 to m . However, considering forests is an obvious approach for searching for either an inductive proof for the graceful tree conjecture or an algorithm for gracefully labelling a tree.

We propose two definitions for what it means for a forest to be graceful. In both cases the vertices are assigned distinct labels from $0, 1, 2, \dots, n - 1$. A forest is *bottom-up* graceful if some such labelling induces a labelling of the edges with distinct values from $1, 2, \dots, m$. A forest is *top-down* graceful if some such labelling induces a labelling of the edges with distinct values from $n - 1, n - 2, n - 3, \dots, n - m$. This talk provides some preliminary results on these two types of labellings of forests including the characterization of an infinite family of forests which are not bottom-up graceful due to a parity condition.

On Friendly Index Sets of Broken Wheels with Three Spokes

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Let G be a graph with vertex set $V(G)$ and edge set $E(G)$, and let A be an abelian group. A labeling $f : V(G) \rightarrow A$ induces an edge vertex labeling $f^* : E(G) \rightarrow A$ defined by $f^*(xy) = f(x) + f(y)$, for each edge $xy \in E(G)$. For $i \in A$, let $v_f(i) = \text{card}\{v \in V(G) : f(v) = i\}$ and $e_f(i) = \text{card}\{e \in E(G) : f^*(e) = i\}$. Let $c(f) = \{|e_f(i) - e_f(j)| : (i, j) \in A \times A\}$. A labeling f of a graph G is said to be A -friendly if $|v_f(i) - v_f(j)| \leq 1$ for all $(i, j) \in A \times A$. If $c(f)$ is a $(0, 1)$ -matrix for an A -friendly labeling f , then f is said to be A -cordial. When $A = \mathbb{Z}_2$, the friendly index set of the graph G is defined as $\{|e_f(0) - e_f(1)| : \text{the vertex labeling } f \text{ is } \mathbb{Z}_2\text{-friendly}\}$. We study friendly index sets of broken wheels with three spokes.

Rainbow Trees in Graphs

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An edge-colored tree T is a rainbow tree if no two edges of T are colored the same. For a nontrivial connected graph G of order n and an integer k with $2 \leq k \leq n$, a k -rainbow coloring of G is an edge coloring having the property that for every set S of k vertices of G , there is a rainbow tree T containing the vertices of S . Related topics are discussed and some results are presented.

Free Mappings and Factorizations of Groups

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We say that a collection of subsets $\alpha = [B_1, \dots, B_k]$ of group G is a *factorization* if $G = B_1 \dots B_k$ and each element of G is expressed in unique way in this product. Group factorization is a topic, that has besides its theoretical beauty practical use in graph theory, coding theory, number theory and cryptography. By using special type of mappings between groups A and B , called *free mappings*, we show a technique for obtaining factorizations of a group G , such that $G \cong A \times B$. Moreover, a simple way for constructing free mappings is provided. There is no limitation on the type of groups A and B , but we found this approach particularly interesting in the case when A and B are abelian groups. It is also considered an interesting joint of free mappings and Rédei's theorem, with number theoretic implication.

Designing Designs

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Interesting groups often occur as automorphism groups of some combinatorial objects. One may use these objects to obtain a representation of the constructed designs. Our combinatorial objects are graphs.

Representing a design graphically may make certain properties of a combinatorial design obvious that otherwise would require tedious reasoning. This holds for symmetries, resolvability, and sub-design inclusion in appropriate cases. We describe how a graphical representation can be achieved in many cases and show several examples.

Some Results on Partitions Inspired by Schur

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Let $f_{(6)}(n)$ denote the number of partitions of the natural number n into parts co-prime to 6. (This function was originally studied by Schur.) We derive two explicit formulas for $f_{(6)}(n)$, one of them in terms of the partition function, $p(n)$. We also derive two recurrences for $f_{(6)}(n)$.

Polygon Dissection and Standard Young Tableaux, Another Bijection

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A diagonalization of a regular n -gon is the placement of chords with the property that each chord has endpoints on non-adjacent vertices of the n -gon and no chord intersects another, except possibly at endpoints. For a fixed regular n -gon and fixed d , we show how to enumerate the number of diagonalizations of the n -gon with d chords by bijecting diagonalizations to a standard Young tableaux of shape $(d+1, d+1, 1^{n-3-d})$. This bijection and others, such as those given by Richard Stanley and I.M.H. Etherington, we summarize and compare, providing several representations for this (originally) geometric object.

On Distance Two Vertex-Magic Graphs

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Given an abelian group A , a graph $G = (V, E)$ is said to have a distance k magic labeling in A if there exists a labeling $l : E(G) \rightarrow A - \{0\}$ such that the induced vertex labeling $l^* : V(G) \rightarrow A$ defined by $l^*(v) = \sum_{e \in E_k(v)} l(e)$ is a constant map, where $E_k(v) = \{e \in E(G) : d(v, e) < k\}$. The set of all $h \in \mathbb{Z}_+$ for which G has a distance two magic labeling in \mathbb{Z}_h is called the distance k magic spectrum of G and is denoted by $\Delta_k M(G)$. In this talk, the distance two magic spectra of certain classes of graphs will be discussed.

More on H-Matchable Trees

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Fix a nontrivial tree H . We call $\{H_1, \dots, H_n\}$ a perfect H -matching of a tree T if and only if H_1, \dots, H_n are subtrees of T where each H_i is isomorphic to H and $\{V(H_1), \dots, V(H_n)\}$ partitions $V(T)$. Then a perfect P_2 -matching of T is a perfect matching of T . A tree is H -matchable if and only if it has a perfect H -matching. We revisit this problem and remind the audience of the formulas presented at this conference in 2006 for the number of labeled H -matchable trees and the degree distribution of a random vertex in a random labeled H -matchable tree. We also prove some asymptotic results about the degree distribution that have not been presented before. Furthermore, we relay the distance distribution for matchable trees and some asymptotic results of this formula as well.

Using Inclusion-Exclusion Principle to Solve Some Problems

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Some Games of odd-even, lottery and murder mysteries are solved using rook polynomials and the Inclusion-Exclusion principle.

Hamiltonian Connected Hourglass free Line Graphs

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The concept of hamiltonian index was first introduced by Chartrand and Wall, who showed that if a connected graph G is not a path, then $L^k(G)$ is defined for any positive integer k . The *hamiltonian index* $h(G)$ of G is the smallest positive integer k such that $L^k(G)$ is hamiltonian. Clark and Wormald extended this idea and introduced Hamiltonian like indices. For a property \mathcal{P} (Hamilton connected, edge-hamiltonian, pancyclic, vertex pancyclic, edge pancyclic, panconnected and fully cycle extendable) and a connected nonempty graph G which is not a path, define the \mathcal{P} -index of G , denoted $\mathcal{P}(G)$, as

$$\mathcal{P}(G) = \begin{cases} \min\{k : L^k(G) \text{ has property } \mathcal{P}\} & \text{if at least one such } k \text{ exists} \\ \infty & \text{otherwise} \end{cases}$$

We summarize some recent results about Hamiltonian like indices of graphs.

On the Minimal Logarithmic Signature Conjecture for Simple Groups of Lie Type

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An ordered tuple $[A_1, A_2, \dots, A_n]$ of ordered subsets of a finite group G is said to be a logarithmic signature (LS) for G , if for each $x \in G$, there exist unique $x_i \in A_i, 1 \leq i \leq n$ such that $x = x_1 x_2 \cdots x_n$. Logarithmic signatures provide several efficient ways to create cryptosystems. The well known minimal logarithmic signature conjecture (MLS conjecture) states that for every finite simple group G there is an optimal logarithmic signature, i.e. an LS of the form $[A_1, A_2, \dots, A_n]$, where each A_i is of size prime or 4. Known results so far imply that an MLS exists for solvable, symmetric, alternating groups and all groups of order $\leq 10^{10}$ with few exceptions. It is also known that MLS's exist for $PSL_n(q)$ when $\gcd(n, q-1) = 1, 4$ or a prime, q a power of a prime. In this talk, the authors develop some new methods for creating such factorizations for the finite groups of Lie type. The methods use the relationship of these groups with the corresponding reductive algebraic groups over algebraically closed fields. In particular, the structure of unipotent and parabolic subgroups and Singer cycles of projective space are used. The logarithmic signatures so obtained will be algorithmically efficient because cyclic subgroups are used to define them. As an application of these methods it is shown that the MLS conjecture is true for $PSL_n(q)$ for all n and q . The applications to other families of classical groups and twisted groups of Lie type will also be discussed.

Minimal Logarithmic Signatures for $Sp_{2n}(q)$ and $PSp_{2n}(q)$

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Logarithmic signatures are a special type of group factorizations which are used to build cryptosystems based on finite groups. Minimal logarithmic signatures are logarithmic signatures of *shortest length*. In this presentation, the authors show that minimal logarithmic signatures exist for the symplectic groups $Sp_{2n}(q)$ and the projective symplectic groups $PSp_{2n}(q)$, for all $n \in \mathbb{N}$ and q a prime power. The structure of the group and the action of the group on the set of isotropic vectors with respect to the bilinear form defining the group, as well as the action of the Singer subgroups on this set have been used to construct minimal logarithmic signatures for these groups. The minimal logarithmic signatures so obtained are algorithmically efficient because cyclic subgroups are used to define them.

Toeplitz and Other Combinatorial Matrices

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In this talk we will discuss some results on banded and other combinatorial matrices. Spectra, factorizations, and number theoretical properties for these matrices will be investigated.

On the Edge-balance Index Sets of Halin Graph of Double Stars

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Let G be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $\mathbb{Z}_2 = \{0, 1\}$. A labeling $f : E(G) \rightarrow \mathbb{Z}_2$ of a graph G is said to be edge-friendly if $\{|e_f(0) - e_f(1)| \leq 1\}$. An edge-friendly labeling f induces a partial vertex labeling $f^+ : V(G) \rightarrow \mathbb{Z}_2$ defined by $f^+(x) = 0$ if the number of edges labeled by 0 incident on x is more than the number of edges labeled by 1 incident on x . Similarly, $f^+(x) = 1$ if the number of edges labeled by 1 incident on x is more than the number of edges labeled by 0 incident on x . $f^+(x)$ is not define if the number of edges labeled by 1 incident on x is equal to the number of edges labeled by 0 incident on x . For $i \in \mathbb{Z}_2$, let $v_f(i) = \text{card}\{v \in V(G) : f^+(v) = i\}$ and $e_f(i) = \text{card}\{e \in E(G) : f(e) = i\}$. The edge-balance index set of the graph G , $\text{EBI}(G)$, is defined as $\{|v_f(0) - v_f(1)| : \text{the edge labeling } f \text{ is edge-friendly}\}$. The edge-balance index sets of the halin graph of double stars are presented in this paper.

Ballot Paths Avoiding Depth Zero Patterns and Finite Operator Calculus

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A ballot path stays weakly above the diagonal $y = x$, starts at the origin, and takes steps from the set $\{\uparrow, \rightarrow\} = \{u, r\}$. A pattern is a finite string made from the same step set; it is also a path. If we consider the number of ballot paths reaching (n, x) as the values of a polynomial sequence $s_n(x)$, the recurrence relation obtained can be transformed into an operator equation. Using Finite Operator Calculus we can find formulas for the polynomial sequence $s_n(x)$. In this paper, we only consider patterns p such that its reverse pattern \tilde{p} is a ballot path. We require this restriction so that the recurrence relation only contains values of the polynomial sequence that correspond to ballot paths, namely $s_n(x)$ with $x \geq n$. For example, the pattern $p = uuurr$ would have the recurrence $s_n(x) = s_{n-1}(x) + s_n(x-1) - s_{n-2}(x-3)$ when $x > n$ and $s_n(x) = s_{n-1}(x)$ when $x = n$, so if we used the first recurrence to define the polynomials, we would be using values below the diagonal that do not correspond to ballot paths. Notice that $\tilde{p} = urrrr$ is not a ballot path. The patterns we consider here are called depth zero. To develop the recursions, we need to investigate the properties of the pattern we wish to avoid.

On $Q(1, 2)$ - and $Q(2, 1)$ -Vertex-graceful Graphs

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For any integer $a, k \geq 1$, a graph G with vertex set $V(G)$ and edge set $E(G)$, $p = |V(G)|$ and $q = |E(G)|$, is said to be $Q(a, k)$ -vertex-graceful (in short $Q(a, k)$ -VG) if there exists a function pair (f, f^+) which assigns integer labels to the vertices and edges, i.e., $f : V(G) \rightarrow \{0, 1, \dots, p-1\}$ and $f^+ : E(G) \rightarrow \{a, a+k, a+2k, \dots, a+(q-1)k\}$ are onto, where $f^+(u, v) = f(u) + f(v)$ for any $(u, v) \in E(G)$.

We determine here classes of graphs that are $Q(2, 1)$ - and $Q(1, 2)$ -vertex-graceful graphs.

Some Combinatorial Problems Over Finite Euclidean and non-Euclidean Spaces

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A classical set of problems in combinatorial geometry deals with the questions of whether a sufficiently large subset of \mathbb{R}^d , \mathbb{Z}^d or \mathbb{F}^d contains a given geometric configuration. Examples are Erdős distance problem, Szemerédi-Trotter theorem and the Furtenberg-Katznelson-Weill theorem. In the finite non-Euclidean spaces, however, the use of known methods, like incidence geometry or Fourier analysis, is nontrivial to the author. We therefore approach the problems using graph theoretic method. Our main tools are graphs associated to the finite Euclidean and non-Euclidean spaces. The advantages of using these graphs are twofold. First, we can reprove and sometimes improve several known results. Secondly, our approach works transparently in the non-Euclidean setting. Due to time constrains, I will only restrict our discussion to the Erdős distance problems and a Furtenberg-Katznelson-Weill type theorem over finite Euclidean and non-Euclidean spaces.

On Triangle-Free Graphs

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We address the question: for what values (n, d) does there exist a regular graph of degree d , on n vertices, that contains no triangle (that is, no subgraph C_3 ? The known results provide upper and lower bounds for n as a function of d , or vice versa) that are close, but there is a small unsettled range.

Two Disjoint Large Cycles in a Graph

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We prove that if G is a graph of order at least $2k$ and the minimum degree of G is at least $k + 1$, then G contains two vertex-disjoint cycles of order at least k .

Symbolic Computation for Integrator Backstepping Control Laws

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Chain integrator Backstepping is a recursive design for systems with nonlinearities not constrained by linear bounds. With backstepping the construction of both feedback control laws and associated Lyapunov functions is systematic. This methodology is important in theory and applications of nonlinear control systems. However, the complexity of the computation of the backstepping control law makes inevitable the use of a computer algebra system. Backstepping of chain of integrators has been studied since 1990. A lemma of chain of integrators was introduced. However, the control law was not given in the lemma. We studied the chain of integrator backstepping and derived the control law. We also use Maple to compute the control law of the nonlinear system. This paper gives control laws of nonlinear systems with chain of integrators. A Maple procedure to compute the control laws is listed. Examples of using the procedure are also given.

On Edge-Balance Indices of Cubic Graphs

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Let G be a finite simple undirected graph with vertex set $V(G)$ and edge set $E(G)$, and let $\mathbf{Z}_2 = \{0, 1\}$. An edge labeling $f : E(G) \rightarrow \mathbf{Z}_2$ of a graph G is called edge-friendly if $|e_f(0) - e_f(1)| \leq 1$, where $e_f(i)$ is the cardinality of the set $\{e \in E(G) : f(e) = i\}$ for $i \in \mathbf{Z}_2$. An edge-friendly labeling f induces a partially defined vertex labeling $f^+ : V(G) \rightarrow \mathbf{Z}_2$ by the following rules: $f^+(v) = 0$ if at the vertex v the number of incident edges labeled 0 is more than the number of incident edges labeled 1, and $f^+(v) = 1$ if at the vertex v the number of incident edges labeled 1 is more than the number of incident edges labeled 0. $f^+(v)$ is not defined if at the vertex v the number of incident edges labeled 1 is equal to the number of incident edges labeled 0. The edge-balance index set of the graph G , denoted by $EBI(G)$, is defined as the set $\{|v_f(0) - v_f(1)| : f \text{ is an edge-friendly edge labeling}\}$, where $v_f(i)$ is the cardinality of the set $\{v \in V(G) : f^+(v) = i\}$ for $i \in \mathbf{Z}_2$. In this article, certain properties of the edge-balance indices of regular graphs are investigated, and in particular the edge-balance index sets of prisms and Möbius ladders are completely determined.

On the Edge-balance Index Sets of $(p, p + 1)$ -graphs

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Let G be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $\mathbb{Z}_2 = \{0, 1\}$. A labeling $f : E(G) \rightarrow \mathbb{Z}_2$ of a graph G is said to be edge-friendly if $\{|e_f(0) - e_f(1)| \leq 1\}$. An edge-friendly labeling f induces a partial vertex labeling $f^+ : V(G) \rightarrow \mathbb{Z}_2$ defined by $f^+(x) = 0$ if the number of edges labeled by 0 incident on x is more than the number of edges labeled by 1 incident on x . Similarly, $f^+(x) = 1$ if the number of edges labeled by 1 incident on x is more than the number of edges labeled by 0 incident on x . $f^+(x)$ is not define if the number of edges labeled by 1 incident on x is equal to the number of edges labeled by 0 incident on x . For $i \in \mathbb{Z}_2$, let $v_f(i) = \text{card}\{v \in V(G) : f^+(v) = i\}$ and $e_f(i) = \text{card}\{e \in E(G) : f(e) = i\}$. The edge-balance index set of the graph G , $\text{EBI}(G)$, is defined as $\{|v_f(0) - v_f(1)| : \text{the edge labeling } f \text{ is edge-friendly.}\}$. The edge-balance index sets of $(p, p + 1)$ -graphs are presented in this paper.

The Hoàng-Reed Conjecture for $\delta^+ = 3$

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Hoàng and Reed have conjectured that a digraph with minimum out-degree δ^+ contains a δ^+ -circuit-forest. A d -circuit-forest is a union of d dicycles C_1, \dots, C_d such that C_k shares at most one vertex with $\bigcup_{i < k} C_i$ for each $k = 2, 3, \dots, d$. Since a d -circuit-forest contains at least $(g - 1)d + 1$ vertices, where g is the girth, the conjecture implies that a digraph with n vertices contains a dicycle of length at most $\lceil n/\delta^+ \rceil$. Thomassen established the Hoàng-Reed Conjecture for $\delta^+ = 2$ in 1985. We have proved the conjecture for $\delta^+ = 3$ by strengthening Thomassen's result for $\delta^+ = 2$ and then examining the global structure of separators in a counter-example for $\delta^+ = 3$.

Random Number Generators: Metrics and Tests for Uniformity and Randomness

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Random number generators are a very small part of any computer simulation project. Yet it is the heart or the engine that drives the rest of the software. Software houses in general fail to understand the complexity involved in building random number generators. Building random number generators with maximum periodicity, providing all the possible permutations with equal probability and at random, requires knowledge of abstract algebra (primitive polynomials and Galois fields), probability theory, statistics, computer simulation, signal processing and computer graphics. In this talk we present a shift register algorithm with perturbation, and most of all we present a number of tests for uniformity and randomness. These tests provide an objective metric for the validity of a random number generator.

Modular Restrictions on Circulant Weighing Matrices

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An $n \times n$ matrix W with entries $(1, -1, 0)$ satisfying $W \cdot W^T = kI$, where I is the identity matrix of order n , is called a weighing matrix of order n and weight k .

A circulant matrix is a square matrix in which each row (except for the first) is a right cyclic shift of its predecessor. The set of all circulant weighing matrices of order n and weight k is denoted by $CW(n, k)$. The ring of all circulant matrices of order n over the integers, \mathbb{Z} , is isomorphic to the quotient ring $R_n = \mathbb{Z}[x]/(x^n - 1)$. A natural isomorphism takes the circulant matrix W with first row $(w_0, w_1, \dots, w_{n-1})$ into the polynomial $w(x) = w_0 + w_1x + \dots + w_{n-1}x^{n-1}$. A polynomial $w(x)$ determines a circulant weighing matrix of weight k if and only if $w(x)w(x_{n-1}) = k$ in R_n . In that case $k = s^2$ where $s = w(1)$.

An integer t in Z_n^* is a multiplier of $w(x)$ from $CW(n, s^2)$ if $w(x) = w(x^t)$. It is known that if $CW(n, s^2) \neq \emptyset$ and $s = p^m$ for some prime p relatively prime to n , then there exist a $w(x)$ in $CW(n, s^2)$ having p as a multiplier.

We use reduction modulo p and the ring structure of $\mathbb{Z}_p[x]/(x^n - 1)$ to obtain certain restrictions on circulant weighing matrices with a multiplier p . We apply these restrictions to derive several new $CW(121, 81)$ with 3-rank 10, 15, 20, and 25 and to prove nonexistence of $CW(273, 121)$, $CW(273, 64)$, and $CW(273, 25)$.

On Product of Factor-Critical Graphs

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A graph G is k -factor-critical if $G - S$ has a perfect matching for all k -subset S of $V(G)$. In this talk, we review the research on factor-criticality and matching-extendability in products of graphs (mainly on Cartesian product). In particular, a recently proved theorem: Cartesian product of an m -factor-critical graph and an n -factor-critical graph is $(m+n+\varepsilon)$ -factor-critical, where $\varepsilon = 0$ if both of m and n are even; otherwise, $\varepsilon = 1$.

Hamiltonian Labelings of Graphs

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A Hamiltonian labeling of a graph G of order n is a vertex labeling c for which the sum of $|c(u) - c(v)|$ and the length of a longest $u - v$ path in G is at least n for every pair u, v of two distinct vertices in G . We investigate the minimum k for which G has a Hamiltonian labeling, all labels of which belong to the set $\{1, 2, \dots, k\}$. Some results involving this concept are described.

Small Even Edge-Cuts of Graphs

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The odd-edge-connectivity of a graph G , denoted by $\lambda_o(G)$, is the size of the smallest odd edge-cut of the graph. We prove that every odd- $(2k+1)$ -connected graph has k edge-disjoint parity subgraphs. We also prove that the flow index of every odd-7-connected graph is less than 4.