The Axiomatic Method

Skolem concluded mathematics is "irreducibly relative." Few followed him. But he drew a widely accepted prior conclusion: There are no non-question-begging categorical axiomatizations of mathematical structures like the natural numbers and the pure sets. But categorical axioms *can* be devised for the natural numbers and *quasicategorical* axioms can be devised for the sets—axioms that guarantee categoricity modulo a choice of height.

Skolem divided logical systems into first-order logic and higher-order and infinitary logic. He proved that first-order logic cannot categorically axiomatize any nontrivial mathematical structure, and he argued that stronger logics presuppose too much. He concluded that axiomatic categorization fails—that is what I contest.

Feferman's full schematic logic suffices for proving the (quasi)categoricity of suitable axiom systems. The proofs are adaptations of some used, though Skolem would argue in a question-begging way, for second-order axiomatizations.

The suggested axioms and theorems in full schematic logic beg no questions. This is the heart of the argument, and the issues are entirely philosophical.

The acceptance of Skolem's criticism of the axiomatic method has led to the belief that, for example, the continuum hypothesis is neither determinately true nor determinately false. In fact, many such problems concerning sets are well-posed problems about a well-characterized system and hence must have determinate answers.

Shaughan Lavine
Department of Philosophy
University of Arizona
P. O. Box 210027
Tucson, Arizona 85721-0027
shaughan@ns.arizona.edu